

CHAPTER I

THE BASIC FORMULA

I. THE AESTHETIC PROBLEM

MANY auditory and visual perceptions are accompanied by a certain intuitive feeling of value, which is clearly separable from sensuous, emotional, moral, or intellectual feeling. The branch of knowledge called aesthetics is concerned primarily with this aesthetic feeling and the aesthetic objects which produce it.

There are numerous kinds of aesthetic objects, and each gives rise to aesthetic feeling which is *sui generis*. Such objects fall, however, in two categories: some, like sunsets, are found in nature, while others are created by the artist. The first category is more or less accidental in quality, while the second category comes into existence as the free expression of aesthetic ideals. It is for this reason that art rather than nature provides the principal material of aesthetics.

Of primary significance for aesthetics is the fact that the objects belonging to a definite class admit of direct intuitive comparison with respect to aesthetic value. The artist and the connoisseur excel in their power to make discriminations of this kind.

To the extent that aesthetics is successful in its scientific aims, it must provide some rational basis for such intuitive comparisons. In fact it is the fundamental problem of aesthetics to determine, within each class of aesthetic objects, those specific attributes upon which the aesthetic value depends.

2. NATURE OF THE AESTHETIC EXPERIENCE

The typical aesthetic experience may be regarded as compounded of three successive phases: (1) a preliminary effort of attention, which is necessary for the act of perception, and which increases in proportion to what we shall call the *complexity* (*C*) of the object; (2) the feeling of value or *aesthetic measure* (*M*) which rewards this effort; and finally (3) a realiza-

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tion that the object is characterized by a certain harmony, symmetry, or order (O), more or less concealed, which seems necessary to the aesthetic effect.

3. MATHEMATICAL FORMULATION OF THE PROBLEM

This analysis of the aesthetic experience suggests that the aesthetic feelings arise primarily because of an unusual degree of harmonious interrelation within the object. More definitely, if we regard M , O , and C as measurable variables, we are led to write

$$M = \frac{O}{C}$$

and thus to embody in a basic formula the conjecture that the aesthetic measure is determined by the density of order relations in the aesthetic object.

The well known aesthetic demand for 'unity in variety' is evidently closely connected with this formula. The definition of the beautiful as that which gives us the greatest number of ideas in the shortest space of time (formulated by Hemsterhuis in the eighteenth century) is of an analogous nature.

If we admit the validity of such a formula, the following mathematical formulation of the fundamental aesthetic problem may be made: *Within each class of aesthetic objects, to define the order O and the complexity C so that their ratio $M = O/C$ yields the aesthetic measure of any object of the class.*

It will be our chief aim to consider various simple classes of aesthetic objects, and in these cases to solve as best we can the fundamental aesthetic problem in the mathematical form just stated. Preliminary to such actual application, however, it is desirable to indicate the psychological basis of the formula and the conditions under which it can be applied.

4. THE FEELING OF EFFORT IN AESTHETIC EXPERIENCE

From the physiological-psychological point of view, the act of perception of an aesthetic object begins with the stimulation of the auditory or visual organs of sense, and continues until this stimulation and the resultant cerebral excitation terminate. In order that the act of perception be successfully performed, there is also required the appropriate field of at-

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tion in consciousness. The attentive attitude has of course its physiological correlative, which in particular ensures that the motor adjustments requisite to the act of perception are effected when required. These adjustments are usually made without the intervention of motor ideas such as accompany all voluntary motor acts, and in this sense are 'automatic.' In more physiological terms, the stimulation sets up a nerve current which, after reaching the cerebral cortex, in part reverts to the periphery as a motor nerve current along a path of extreme habituation, such as corresponds to any automatic act.

Now, although these automatic adjustments are made without the intervention of motor ideas, nevertheless there is a well-known feeling of effort or varying tension while the successive adjustments are called for and performed. This constitutes a definite and important part of the general feeling characteristic of the state of attention. The fact that interest of some kind is almost necessary for sustained attention would seem to indicate that this feeling has not a positive (pleasurable) tone but rather a negative one. Furthermore, if we bear in mind that the so-called automatic acts are nothing but the outcome of unvarying voluntary acts habitually performed, we may reasonably believe that there remain vestiges of the motor ideas originally involved, and that it is these which make up this feeling of effort.

From such a point of view, the feeling of effort always attendant upon perception appears as a summation of the feelings of tension which accompany the various automatic adjustments.

5. THE PSYCHOLOGICAL MEANING OF 'COMPLEXITY'

Suppose that A, B, C, \dots are the various automatic adjustments required, with respective indices of tension a, b, c, \dots , and that these adjustments A, B, C, \dots take place r, s, t, \dots times respectively. Now it is the feeling of effort or tension which is the psychological counterpart of what has been referred to as the complexity C of the aesthetic object. In this manner we are led to regard the sum of the various indices as the measure of complexity, and thus to write

$$C = ra + sb + tc + \dots$$

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A simple illustration may serve to clarify the point of view. Suppose that we fix attention upon a convex polygonal tile. The act of perception involved is so quickly performed as to seem nearly instantaneous. The feeling of effort is almost negligible while the eye follows the successive sides of the polygon and the corresponding motor adjustments are effected automatically. Nevertheless, according to the point of view advanced above, there is a slight feeling of tension attendant upon each adjustment, and the complexity *C* will be measured by the number of sides of the polygon.

Perhaps a more satisfying illustration is furnished by any simple melody. Here the automatic motor adjustments necessary to the act of perception are the incipient adjustments of the vocal cords to the successive tones. Evidently in this case the complexity *C* will be measured by the number of notes in the melody.

6. ASSOCIATIONS AND AESTHETIC FEELING

Up to this point we have only considered the act of perception of an aesthetic object as involving a certain effort of attention. This feeling of effort is correlated with the efferent part of the nerve current which gives rise to the required automatic motor adjustments, and has no direct reference to aesthetic feeling.

For the cause (physiologically speaking) of aesthetic feeling, we must look to that complementary part of the nerve current which, impinging on the auditory and visual centers, gives rise to sensations derived from the object, and, spreading from thence, calls various associated ideas with their attendant feelings into play. These sensations, together with the associated ideas and their attendant feelings, constitute the full perception of the object. It is in these associations rather than in the sensations themselves that we shall find the determining aesthetic factor.

In many cases of aesthetic perception there is more or less complete identification of the percipient with the aesthetic object. This feeling of 'empathy,' whose importance has been stressed by the psychologist Lipps,* contributes to the enhancement of the aesthetic effect. Similarly,

* *Ästhetik: Psychologie des Schönen und der Kunst*, Hamburg and Leipzig, vol. 1 (1903), vol. 2 (1906).

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actual participation on the part of the percipient, as in the case of singing a tune as well as hearing it, will enhance the effect.

7. THE INTUITIVE NATURE OF SUCH ASSOCIATIONS

Mere verbal associations are irrelevant to the aesthetic experience. In other words, aesthetic associations are *intuitive* in type.

When, for instance, I see a symmetrical object, I feel its pleasurable quality, but do not need to assert explicitly to myself, "How symmetrical!" This characteristic feature may be explained as follows. In the course of individual experience it is found generally that symmetrical objects possess exceptional and desirable qualities. Thus our own bodies are not regarded as perfectly formed unless they are symmetrical. Furthermore, the visual and tactual technique by which we perceive the symmetry of various objects is uniform, highly developed, and almost instantaneously applied. It is this technique which forms the associative 'pointer.' In consequence of it, the perception of any symmetrical object is accompanied by an intuitive aesthetic feeling of positive tone.

It would even seem to be almost preferable that no verbal association be made. The unusual effectiveness of more or less occult associations in aesthetic experience is probably due to the fact that such associations are never given verbal reference.

8. THE RÔLE OF SENSUOUS FEELING

The typical aesthetic perception is primarily of auditory or visual type, and so is not accompanied by stimulation of the end-organs of the so-called lower senses. Thus the sensuous feeling which enters will be highly refined. Nevertheless, since sensuous feeling with a slight positive tone ordinarily accompanies sensations of sight and of sound, it might appear that such sensuous feeling requires some consideration as part of the aesthetic feeling. Now, in my opinion, this component can be set aside in the cases of most interest just because the positive tone of sensuous feeling is always present, and in no way differentiates one perception from another.

For example, all sequences of pure musical tones are equally agreeable as far as the individual sensations are concerned. Yet some of these se-

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quences are melodic in quality, while others are not. Hence, although the agreeableness of the individual sounds forms part of the tone of feeling, we may set aside this sensuous component when we compare the melodic quality of various sequences of musical tones.

To support this opinion further, I will take up briefly certain auditory facts which at first sight appear to be in contradiction with it.

If a dissonant musical interval, such as a semitone, is heard, the resultant tone of feeling is negative. Similarly, if a consonant interval like the perfect fifth is heard, the resultant tone of feeling is positive. But is not the sensation of a dissonant interval to be considered a single auditory sensation comparable with that of a consonant interval, and is it not necessary in this case at least to modify the conclusion as to the constancy of the sensuous factor?

In order to answer this question, let us recall that musical tones, as produced either mechanically or by the human voice, contain a pure fundamental tone of a certain frequency of vibration and pure overtones of double the frequency (the octave), of triple the frequency (the octave of the perfect fifth), etc.; here, with Helmholtz, we regard a pure tone as the true individual sensation of sound. Thus 'association by contiguity' operates to connect any tone with its overtones.

If such be the case, a dissonant interval, being made up of two dissociated tones, may possess a negative tone of feeling on account of this dissociation; while the two constituent tones of a consonant interval, being connected by association through their overtones, may possess a positive tone of feeling in consequence. Hence the obvious difference in the aesthetic effect of a consonant and a dissonant musical interval can be explained on the basis of association alone.

9. FORMAL AND CONNOTATIVE ASSOCIATIONS

It is necessary to call attention to a fundamental division of the types of associations which enter into the aesthetic experience.

Certain kinds of associations are so simple and unitary that they can be at once defined and their rôle can be ascertained with accuracy. On the other hand, there are many associations, of utmost importance from the aesthetic point of view, which defy analysis because they touch our

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experience at so many points. The associations of the first type are those such as symmetry; an instance of the second type would be the associations which are stirred by the *meaning* of a beautiful poem.

For the purpose of convenient differentiation, associations will be called 'formal' or 'connotative' according as they are of the first or second type. There will of course be intermediate possibilities.

More precisely, formal associations are such as involve reference to some simple physical property of the aesthetic object. Two simple instances of these are the following:

rectangle in vertical position → symmetry about vertical;
interval of note and its octave → consonance.

There is no naming of the corresponding property, which is merely pointed out, as it were, by the visual or auditory technique involved.

All associations which are not of this simple formal type will be called connotative.

IO. FORMAL AND CONNOTATIVE ELEMENTS OF ORDER

The property of the aesthetic object which corresponds to any association will be called an 'element of order' in the object; and such an element of order will be called formal or connotative according to the nature of the association. Thus a formal element of order arises from a simple physical property such, for instance, as that of consonance in the case of a musical interval or of symmetry in the case of a geometrical figure.

It is not always the case that the elements of order and the corresponding associations are accompanied by a positive tone of feeling. For example, sharp dissonance is to be looked upon as an element of order with a negative tone of feeling.

II. TYPES OF FORMAL ELEMENTS OF ORDER

The actual types of formal elements of order which will be met with are mainly such obvious positive ones as repetition, similarity, contrast, equality, symmetry, balance, and sequence, each of which takes many forms. These are in general to be reckoned as positive in their effect.

Furthermore there is a somewhat less obvious positive element of order, due to suitable centers of interest or repose, which plays a rôle. For

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example, a painting should have one predominant center of interest on which the eye can rest; similarly in Western music it is desirable to commence in the central tonic chord and to return to this center at the end.

On the other hand, ambiguity, undue repetition, and unnecessary imperfection are formal elements of order which are of strongly negative type. A rectangle nearly but not quite a square is unpleasantly ambiguous; a poem overburdened with alliteration and assonance fatigues by undue repetition; a musical performance in which a single wrong note is heard is marred by the unnecessary imperfection.

12. THE PSYCHOLOGICAL MEANING OF 'ORDER'

We are now prepared to deal with the order O of the aesthetic object in a manner analogous to that used in dealing with the complexity C .

Let us suppose that associations of various types L, M, N, \dots take place with respective indices of tone of feeling l, m, n, \dots . In this case the indices may be positive, zero, or negative, according as the corresponding tones of feeling are positive, indifferent, or negative. If the associations L, M, N, \dots occur u, v, w, \dots times respectively, then we may regard the total tone of feeling as a summational effect represented by the sum $ul + vm + \dots$.

This effect is the psychological counterpart of what we have called the order O of the aesthetic object, inasmuch as L, M, N, \dots correspond to what have been termed the elements of order in the aesthetic object. Thus we are led to write

$$O = ul + vm + wn + \dots$$

By way of illustration, let us suppose that we have before us various polygonal tiles in vertical position. What are the elements of order and the corresponding associations which determine the feeling of aesthetic value accompanying the act of perception of such a tile? Inasmuch as a detailed study of polygonal form is made in the next chapter, we shall merely mention three obvious positive elements of order, without making any attempt to choose indices. If a tile is symmetric about a vertical axis, the vertical symmetry is felt pleasantly. Again, a tile may have symmetry of rotation; a square tile, for example, has this property, for it

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can be rotated through a right angle without affecting its position. Such symmetry of rotation is also appreciated immediately. Lastly, if the sides of a tile fall along a rectangular network, as in the case of a Greek cross, the relation to the network is felt agreeably.

13. THE CONCEPT OF AESTHETIC MEASURE

The aesthetic measure M of a class of aesthetic objects is primarily any quantitative index of their comparative aesthetic effectiveness.

It is impossible to compare objects of different types, as we observed at the outset. Who, for instance, would attempt to compare a vase with a melody? In fact, for comparison to be possible, such classes must be severely restricted. Thus it is futile to compare a painting in oils with one in water colors, except indirectly, by the comparison of each with the best examples of its type; to be sure, the two paintings might be compared, in respect to composition alone, by means of photographic reproduction. On the other hand, photographic portraits of the same person are readily compared and arranged in order of preference.

But even when the class is sufficiently restricted, the preferences of different individuals will vary according to their taste and aesthetic experience. Moreover the preference of an individual will change somewhat from time to time. Thus such aesthetic comparison, of which the aesthetic measure M is the determining index, will have substantial meaning only when it represents the normal or average judgment of some selected group of observers. For example, in the consideration of Western music it would be natural to abide by the consensus of opinion of those who are familiar with it.

Consequently the concept of aesthetic measure M is applicable only if the class of objects is so restricted that direct intuitive comparison of the different objects becomes possible, in which case the arrangement in order of aesthetic measure represents the aesthetic judgment of an idealized 'normal observer.'

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If our earlier analysis be correct, it is the intuitive estimate of the amount of order O inherent in the aesthetic object, as compared with its complexity C , from which arises the derivative feeling of the aesthetic

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measure M of the different objects of the class considered. We shall first make an argument to this effect on the basis of an analogy, and then proceed to a more purely mathematical argument.

The analogy will be drawn from the economic field. Among business enterprises of a single definite type, which shall be held the most successful? The usual answer would take the following form. In each business there is involved a certain investment i and a certain annual profit p . The ratio p/i , which represents the percentage of interest on the investment, is regarded as the economic measure of success.

Similarly in the perception of aesthetic objects belonging to a definite class, there is involved a feeling of effort of attention, measured by C , which is rewarded by a certain positive tone of feeling, measured by O . It is natural that reward should be proportional to effort, as in the case of a business enterprise. By analogy, then, it is the ratio O/C which best represents the aesthetic measure M .

15. A MATHEMATICAL ARGUMENT *

More mathematically, but perhaps not more convincingly, we can argue as follows. In the first place it must be supposed that if two objects of the class have the same order O and the same complexity C , their aesthetic measures are to be regarded as the same. Hence we may write

$$M = f(O, C)$$

and thus assert that the aesthetic measure depends functionally upon O and C alone.

It is obvious that if we increase the order without altering the complexity, or if we diminish the complexity without altering the order, the value of M should be increased. But these two laws do not serve to determine the function f .

In order to do so, we imagine the following hypothetical experiment. Suppose that we have before us a certain set of k objects of the class, all having the same order O and the same complexity C , and also a second set of k' objects of the class, all having the order O' and complexity C' . Let us choose k and k' so that $k'C'$ equals kC .

* This mathematical section may be omitted.

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Now proceed as follows. Let all of the first set of objects be observed, one after the other; the total effort will be measured by kC of course, and the total tone of aesthetic feeling by kO . Similarly let all of the second set be observed. The effort will be the same as before, since kC equals kC ; and the total tone of feeling will be measured by $k'O'$.

If the aesthetic measure of the individual objects of the second class is the same as of the first, it would appear inevitable that the total tone of feeling must be the same in both cases, so that $k'O'$ equals kO . With this granted, we conclude at once that the ratios O'/C' and O/C are the same. In consequence the aesthetic measure only depends upon the ratio O to C :

$$M = f \left(\frac{O}{C} \right).$$

The final step can now be taken. Since it is not the actual numerical magnitude of f that is important but only the relative magnitude when we order according to aesthetic measure, and since M must increase with O/C , we can properly define M as equal to the ratio of O to C .

It is obvious that the aesthetic measure M as thus determined is zero ($M = 0$) when the tone of feeling due to the associated ideas is indifferent.

16. THE SCOPE OF THE FORMULA

As presented above, the basic formula admits of theoretic application to any properly restricted class of aesthetic objects.

Now it would seem not to be difficult in any case to devise a reasonable and simple measure of the complexity C of the aesthetic objects of the class. On the other hand, the order O must take account of all types of associations induced by the objects, whether formal or connotative; and a suitable index is to be assigned to each. Unfortunately the connotative elements of order cannot be so treated, since they are of inconceivable variety and lie beyond the range of precise analysis.

It is clear then that complete quantitative application of the basic formula can only be effected when the elements of order are mainly formal. Of course it is always possible to consider the formula only in so far as the formal elements of order are concerned, and to arrive in this way at a partial application.

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Consequently our attention will be directed almost exclusively towards the formal side of art, to which alone the basic formula of aesthetic measure can be quantitatively applied. Our first and principal aim will be to effect an analysis in typical important cases of the utmost simplicity. From the vantage point so reached it will be possible to consider briefly more general questions. In following this program, there is of course no intention of denying the transcendent importance of the connotative side in all creative art.

17. A DIAGRAM

The adjoining diagram with the attached legend may be of assistance in recalling the above analysis of the aesthetic experience and the basic aesthetic formula to which it leads.

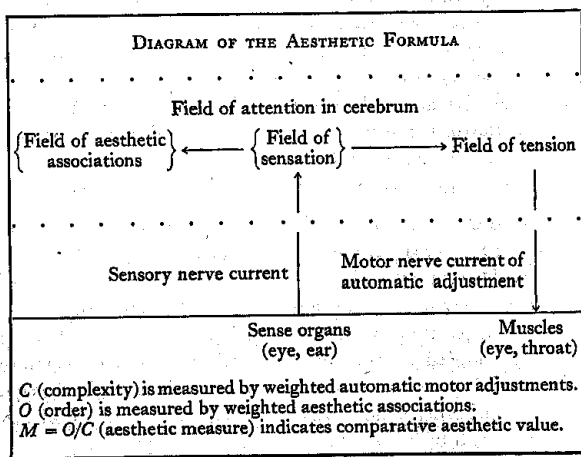


FIGURE 1

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18. THE METHOD OF APPLICATION

Even in the most favorable cases, the precise rules adopted for the determination of O , C , and thence of the aesthetic measure M , are necessarily empirical. In fact the symbols O and C represent social values, and share in the uncertainty common to such values. For example, the 'purchasing power of money' can only be determined approximately by means of empirical rules, and yet the concept involved is of fundamental economic importance.

At the same time it should be added that this empirical method seems to be the only one by which concepts of this general category can be approached scientifically.

We shall endeavor at all times to choose formal elements of order having unquestionable aesthetic importance, and to define indices in the most simple and reasonable manner possible. The underlying facts have to be ascertained by the method of direct introspection.

In particular we shall pay attention to the two following *desiderata*:

As far as possible these indices are to be taken as equal, or else in the simplest manner compatible with the facts.

The various elements of order are to be considered only in so far as they are logically independent. If, for example, $a = b$ is an equality which enters in O , and if $b = c$ is another such equality, then the equality $a = c$ will not be counted separately.

CHAPTER II

POLYGONAL FORMS

I. POLYGONS AS AESTHETIC OBJECTS

POLYGONS, considered merely in their aspect of geometric form, have a definite, if elementary, aesthetic appeal. This fact has always been recognized, and is borne out by their wide use for decorative purposes in East and West (see Plate I opposite). Moreover such polygonal forms can be intuitively compared with one another with respect to aesthetic quality. For example, Alison says: * "An Equilateral Triangle is more beautiful than a Scalene or an Isosceles, a Square than a Rhombus, an Hexagon than a Square, an Ellipse than a Parabola, a Circle than an Ellipse; because the number of their uniform parts are greater, and their Expression of Design more complete."

Evidently then polygonal forms constitute a class of aesthetic objects of the utmost simplicity, which have the further advantage of being relatively free from connotative elements of order. It is for these reasons that the first application of the general theory is made to polygonal forms.

2. PRELIMINARY REQUIREMENTS

According to the general theory, it is necessary to select some specific type of representation of the polygons. For the sake of definiteness we shall have in mind porcelain tiles of polygonal shape, and alike in size, color, and material. In this way the class of objects to be considered is precisely defined. Other classes of polygonal objects might be considered, such for instance as precious stones cut in polygonal form. But it is evident that then various factors other than form would be likely to enter, such as the brilliancy of the reflected light. Thus the choice of tiles is advantageous, since these differ from one another only in their aspect of geometric form.

* *Essays on the Nature and Principles of Taste*, Edinburgh (1790).

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A further requirement must be imposed in order to fix the psychological condition of the 'normal observer.' Such a polygonal tile produces a somewhat different impression when it is seen upon a table than when it is seen in vertical position. In fact a tile lying upon the table would be viewed from various directions, while one in vertical position is seen in a single orientation. Therefore it is desirable to think of such tiles as in vertical position. In general the selected orientation will of course be the best one. Perhaps the actual use for decorative purposes which most nearly conforms to these conditions is that in which identical porcelain tiles appear at regular intervals in the same orientation along a stuccoed wall.

Just as in other aesthetic fields, a certain degree of familiarity with the various types of objects involved is required before the aesthetic judgment becomes certain. It is hardly necessary to observe that when novel polygons, pleasing in themselves, are seen for the first time, they take higher rank than they do subsequently, just because of this novelty.

It is clear that when these requirements are satisfied, the aesthetic problem of polygonal form becomes a legitimate one in the sense of the preceding chapter.

3. SYMMETRY OF POLYGONS

It is desirable at the outset to obtain a clear idea of the types of symmetry which occur in polygonal forms, since such symmetry is evidently of fundamental importance from the aesthetic point of view.

There are two types of symmetry which a polygon may possess. The first and simpler is that of 'symmetry about a line' in its plane, called the 'axis of symmetry.' If the figure be rotated about this line through an angle of 180° , it returns as a whole to its initial position, while the individual points are transferred to the symmetric points on the other side of the axis of symmetry.

In a square the two diagonals are axes of symmetry as well as the two lines through its center which are parallel to a pair of sides. In a rectangle there are only two axes of symmetry, namely the two lines through its center parallel to a pair of sides. A parallelogram is not symmetric about any line through its center.

The second type is that of 'rotational symmetry about a point' in the

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plane of the polygon, called the 'center of symmetry.' If the polygon be rotated in the plane through a certain angle about this point, it returns as a whole to its initial position, while the individual points are rotated through this angle about the point. The angle in question will be called the 'angle of rotation.' Here the axis of rotation is a line perpendicular to the plane of the polygon at the center of symmetry.

The square possesses such rotational symmetry about its center, and the angle of rotation is clearly 90° or one quarter of a complete revolution. Similarly the rectangle and parallelogram have rotational symmetry with 180° as angle of rotation.

An isosceles triangle illustrates the fact that a polygon may possess symmetry about an axis without having rotational symmetry; and the case of the parallelogram shows that it may possess rotational symmetry without having symmetry about an axis.

By the angle of rotation we mean of course the least such angle. Suppose that q successive rotations through this least angle effect one complete revolution. The least angle of rotation is then $360^\circ/q$. But twice this angle, three times this angle, and, more generally, any multiple of it, are admissible angles also.

When q is an even number, $q/2$ rotations through the least angle will amount to a half revolution or 180° . A polygon with an admissible angle of rotation of 180° is said to possess 'central symmetry.'

In this case any point of the polygon is paired with an opposite corresponding point, so that the line joining the two points is bisected by the center of symmetry. Such central symmetry exists if, and only if, an even number of rotations, q , through the angle of rotation is required before a complete revolution is effected. Evidently the rectangle and parallelogram with q equal to 2, and the square with q equal to 4, have central symmetry, while the equilateral triangle with q equal to 3 has not, in accordance with the statement just made.

It is clear that the above definitions apply to all plane geometrical figures as well as to polygons. In the exceptional case of the circle, every line through the center is an axis of symmetry, and every angle is an admissible angle of rotation. This is not true of any plane figure other than the circle.

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4. ISOSCELES AND EQUILATERAL TRIANGLES

What then are the principal aesthetic factors * involved in polygonal form? Once these have been determined, we shall be able to define the corresponding elements of order in O , and the complexity C , and thus arrive at an appropriate aesthetic measure. We shall begin with the simplest class of polygons, namely the triangles.

Now triangles are usually classified as being either isosceles or scalene. Clearly the isosceles triangles, inclusive of the equilateral triangle, are more interesting from the aesthetic point of view. The usual orientation of an isosceles triangle is one in which the two equal sides are inclined at the same angle to the vertical, while the triangle rests on the third side. This is the case for each of the triangles (a), (b), (c) of the adjoining figure.

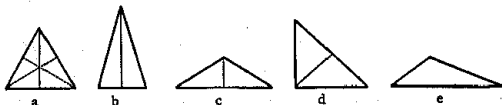


FIGURE 2

However, if these triangles are inverted, the equal sides will again be inclined at the same angle to the vertical. It is readily verified that this reversed orientation is also satisfactory. With these two orientations only do we obtain symmetry about a vertical axis. This is obviously a *desideratum* of prime importance.

The observation of symmetry about a vertical axis occurs constantly in everyday experience. Let us recall, for example, how quickly we become aware of any variation from symmetry in the human face. The association of vertical symmetry is intuitive and pleasing.

If the isosceles triangle (b) be made to rest upon one of the two equal sides, there still remains the feeling that the triangle is in equilibrium, although the symmetry about the vertical axis is thereby destroyed. It will be observed furthermore that the symmetry about the inclined axis is scarcely noted by the eye. Thus the triangle in its new orientation makes

* The term 'aesthetic factor' will be used in a non-technical sense.

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much the same impression as any scalene triangle which rests upon a horizontal side (compare with (e)). The indifference of the eye to such an inclined axis of symmetry is also evidenced by the isosceles right triangle (d) with one of its equal sides horizontal.

On the other hand, if the isosceles triangle (b) be given any orientation whatsoever other than the two with vertical symmetry and the third just considered, there is a definite and displeasing lack of equilibrium.

For the isosceles triangle (c), however, there are only two orientations in which it seems to have full equilibrium, namely the two with vertical symmetry. In fact if the triangle (c) be made to rest upon one of its equal sides, the center of area falls so far to left or right as to give rise to the feeling that the equilibrium is not complete. The 'center of area' of any polygon is defined as that point of support about which it balances in a horizontal plane. Of course the polygon is assumed to be of uniform surface-density.

The association of equilibrium is also developed in our everyday experience.

Among the various shapes of isosceles triangles, the equilateral triangle (a) stands out as possessing peculiar interest because of its rotational symmetry. If such an equilateral triangle be set in a position which is not symmetrical about a vertical axis, all the pleasure in this symmetric quality disappears. However, once the favorable orientation is taken, the rotational symmetry is appreciated, largely by means of the three axes of symmetry. In the equilateral triangle the center of symmetry (and area) is the point of intersection of the three axes of symmetry, and the angle of rotation is 120° or one third of a complete revolution.

Associative reference to rotational symmetry often occurs in everyday experience. The form of the circle may perhaps be regarded as inducing this reference most completely.

Among the isosceles triangles which are not equilateral, there seems to be little to choose in respect to aesthetic merit. It does not appear to be a matter of importance whether the angle between the two equal sides is acute as in (b), obtuse as in (c), or a right angle. Of course when such a triangle is used in conjunction with other geometrical forms, this is no longer necessarily the case.

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5. SCALENE TRIANGLES

The scalene triangles are readily disposed of. The best position is one in which the triangle rests upon a horizontal side long enough for the triangle to be in complete equilibrium. The right triangle with vertical and horizontal sides is obviously the best among the scalene triangles (note the triangle (d)). From our point of view this is because an unfavorable factor enters into the general scalene triangle of type (e) due to the presence of *three* unrelated directions.

6. CONCLUSIONS CONCERNING TRIANGULAR FORMS

Thus the various types of triangles in a vertical plane can be grouped in the following five classes according to order of aesthetic value: (1) the equilateral triangle with vertical axis of symmetry; (2) the isosceles triangle with vertical axis of symmetry; (3) the right triangle with vertical and horizontal sides; (4) any other triangle resting upon a sufficiently long horizontal side to ensure the feeling of complete equilibrium; (5) any triangle which lacks equilibrium. The triangles of the first two classes are definitely pleasing; those of the third class are perhaps to be considered indifferent in quality; and those of the fourth and fifth classes are displeasing. Since it is a natural requirement that the best orientation of any triangle be selected, the fourth class will contain all of the scalene triangles without a right angle, and the fifth class will scarcely enter into consideration.

It has been tacitly assumed in the above analysis of triangular form that no side of the triangle is extremely small in comparison to the other two sides, and that no angle is very small or very near to 180° . These are obvious prerequisites if the triangle is to be characteristic. If they are not satisfied, the triangle approximates in form to a straight line and the effect of ambiguity is definitely disagreeable. Likewise, when the triangle is very nearly but not quite isosceles, or very nearly but not quite equilateral, there is produced a feeling of ambiguity.

We are now in a position to list the aesthetic factors that have been so far encountered: vertical symmetry (+), symmetry about an inclined axis (o), equilibrium (+), rotational symmetry (+), diversity of direc-

AESTHETIC MEASURE

tions (-), small side (-), small angle or angle nearly 180° (-), other ambiguity (-).

Here and later we use the symbol (+) to indicate that the corresponding factor is positive (that is, increases aesthetic value), the symbol (o) to indicate that it is without substantial effect, and the symbol (-) to indicate that it is negative (that is, diminishes aesthetic value). It is to be observed that, among the three positive factors, that of equilibrium is regarded as having a negative aspect also, arising from lack of equilibrium.

7. PLATO'S FAVORITE TRIANGLE

The classification of the various forms of triangles given above takes only formal aesthetic factors into account. How completely such a classi-

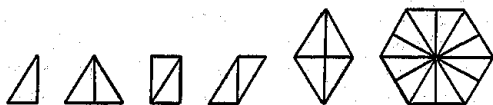


FIGURE 3

fication can be upset by the introduction of fortuitous connotative factors is easily illustrated.

Plato in the *Timaeus* says: * "Now, the one which we maintain to be the most beautiful of all the many figures of triangles (and we need not speak of the others) is that of which the double forms a third equilateral triangle." The context makes perfectly clear in what sense his statement is to be interpreted: If one judges the beauty of a triangle by its power to furnish other interesting geometrical figures by combination, there is no other triangle comparable with this favorite triangle of Plato. For out of it can be built (see Figure 3) the equilateral triangle, rectangle, parallelogram, diamond, and regular hexagon among polygons, as well as three of the five regular solids. This power in combination was peculiarly significant to Plato, who valued it for purposes of cosmological speculation. It was on such a mystical view that he based his aesthetic preference for this particular triangle.

* Translation by Jowett.

POLYGONAL FORMS

8. THE SCALENE TRIANGLE IN JAPANESE ART

It is well known that the Japanese prefer to use asymmetric form rather than the too purely symmetric. Indeed in all art, whether Eastern or Western, irrelevant symmetry is tiresome.

In particular it has been said that composition in Japanese painting is based upon the scalene triangle. Is this fact in agreement with the classification effected above, which concedes aesthetic superiority to the isosceles and in particular to the equilateral triangle? The answer seems to be plain: When used as an element of composition in painting, the isosceles triangle may introduce an adventitious element of symmetry. But, in the more elementary question of triangular form *per se*, the equilateral and isosceles triangles are superior to the scalene triangle.

Recently while in Japan I was fortunate enough to be able to ask one of the greatest Japanese painters, Takeuchi Seiho, if this were not the case, and he told me that the same opinion would doubtless be held in Japan.

9. SYMMETRIC QUADRILATERALS (FIRST TYPE)

Let us turn next to the consideration of the form of quadrilaterals, and let us examine those first in which there is symmetry about a vertical

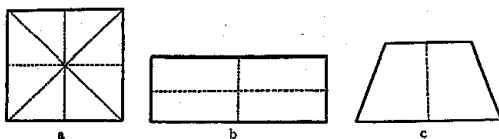


FIGURE 4

axis. There are two types. In the first, at least one side of the quadrilateral intersects the axis of symmetry; evidently such a side must be perpendicular to the axis of symmetry. Furthermore there must then be a second opposite side which is also perpendicular to the axis. Thus the general possibility is that of a symmetric trapezoid given by (c) of the figure above. This trapezoid may, however, take the form of a rectangle or square, illustrated by (b) and (a) respectively. It is to be observed that the

AESTHETIC MEASURE

rectangle possesses a further horizontal axis of symmetry, while the square possesses not only horizontal and vertical axes of symmetry but also two axes inclined at 45° to the horizontal direction. Likewise both rectangle and square have rotational symmetry, the angles of rotation being 180° and 90° respectively.

Corresponding to the degree of symmetry involved, we should expect to find the square to be the best in form, the rectangle excellent, and both superior in aesthetic quality to the symmetrical trapezoid. Such a relative rating coincides, I believe, with the facts.

It has been claimed sometimes that the rectangle is a form superior to the square, and even that certain rectangles such as the 'Golden Rectangle' surpass all others. We shall consider later (sections 14, 15) in what sense, if any, such assertions can be valid.

10. SYMMETRIC QUADRILATERALS (SECOND TYPE)

In the second type of symmetry about the vertical axis, the quadrilateral has two of its vertices on the axis of symmetry, but none of the

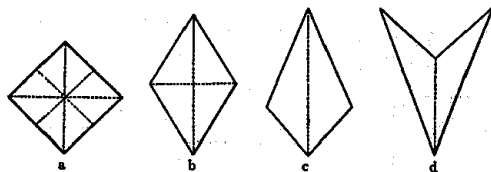


FIGURE 5

sides intersect the axis. Here the general possibility is indicated by (c) and (d) of the next figure (Figure 5) in which the deltoid (c) is convex, while (d) is re-entrant. The first of these may, however, take the form of an equilateral quadrilateral or diamond as in (b), or even of the square (a) with sides inclined at 45° to the horizontal direction.

Of the two general cases represented by the quadrilaterals (c) and (d) it is clear that the convex type (c) is definitely superior to the alternative re-entrant quadrilateral (d). The latter quadrilateral suggests a triangle from which a triangular niche has been removed.

POLYGONAL FORMS

It is not the mere fact that the quadrilateral is re-entrant which is decisively unfavorable. Consider, for example, the ordinary six-pointed star having the same outline as the familiar hexagram. This star is evidently highly pleasing in form, and yet is re-entrant. It will be noted, however, that every side, although of re-entrant type, is supported by another side which lies in the same straight line, while this is not true of the re-entrant sides of the quadrilateral above.

A re-entrant side will be termed 'supported' or 'unsupported' according as another side of the polygon does or does not lie in the same straight line.

In the comparison of quadrilaterals of types (a)-(d) we find, as we should expect, that the square (a) and the diamond (b) in the orientations indicated are markedly superior to the quadrilaterals (c) and (d) already discussed. However, it seems difficult to say whether or not the square so situated is better in form than the diamond, despite the fact that, on the score of symmetry alone, the square holds higher rank.

As far as I can analyze my own impressions, I am led to the following explanation of this aesthetic uncertainty: For me and many other persons the orientation of the square with sides vertical and horizontal is superior to the orientation with inclined sides. This superiority agrees with the theory of aesthetic measure, according to which the square in horizontal position has the highest rating of all polygonal forms ($M = 1.50$), while the square in the inclined position, together with the rectangle in horizontal position, come next ($M = 1.25$). Hence there arises a feeling of 'unnecessary imperfection' (Chapter I, section 11) when the square is in the inclined orientation, just because it would be *so easy* to alter it for the better. As soon as this association, which is really irrelevant, is abstracted from, the inclined square (a) will be found, I believe, to be superior to the diamond (b).

II. SYMMETRIC QUADRILATERALS (THIRD TYPE)

There remain for discussion those quadrilaterals which are not symmetric about an axis. Here, as in the case of the scalene triangle, attention may be limited to cases in which the quadrilateral rests on a sufficiently long horizontal side so that it appears to be in complete equilibrium.

AESTHETIC MEASURE

It is readily seen that the only quadrilaterals of this type which possess rotational symmetry are the parallelograms, illustrated by (a) in the adjoining figure. Evidently the angle of rotation for a parallelogram is 180° , so that it possesses central symmetry.

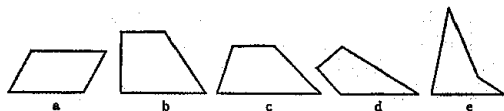


FIGURE 6

The parallelogram is the simplest polygon possessing rotational symmetry but not symmetry about the axis. It is more satisfactory in form than the remaining asymmetric quadrilaterals, just because of its symmetric character.

12. ASYMMETRIC QUADRILATERALS

Among the asymmetric quadrilaterals, represented by (b), (c), (d), (e) in the figure, the right-angle and general trapezoids with two sides parallel, as in (b) and (c), stand first.

The associative reference to 'parallelism' occurs frequently in everyday experience. The factor of parallelism is closely correlated with that of 'diversity of directions,' already listed. For, the more sides are parallel, the less will be the diversity of directions.

After the trapezoids in aesthetic quality follow the general convex quadrilateral (d), and finally the re-entrant case (e). It will be observed that the presence of an isolated right angle or of two equal sides, as in (d), is without noticeable influence.

13. CONCLUSIONS CONCERNING QUADRILATERAL FORMS

We have now examined the types of quadrilateral forms and have arranged those of each type in order of aesthetic merit. It remains to compare briefly those of different types.

The usual order of preference appears to be in the following groups of diminishing aesthetic value: the square; the rectangle; the diamond; the symmetric trapezoid, the deltoid, and the parallelogram; the re-

POLYGONAL FORMS

entrant quadrilateral symmetric about an axis; the right-angle trapezoid; the remaining convex quadrilaterals and re-entrant quadrilaterals without symmetry. Of these, the last group is definitely unsatisfactory. It is assumed here that the quadrilaterals are placed in the best vertical orientation. This relative arrangement is the same as that assigned by the theory of the present chapter. We shall not attempt at this juncture to compare quadrilaterals and triangles.

Thus there are two kinds of aesthetic factors brought to light by our examination of quadrilaterals. The first is of negative type and is occasioned by the fact that the quadrilateral is re-entrant. Further insight into the nature of this factor will be obtained as we proceed. The second factor is connected with the parallelism of sides. As we have observed, it is more convenient to regard this second factor in its negative aspect, when it appears as that of diversity of directions.

It has been seen also that the mere equality of sides is an indifferent factor for quadrilaterals, as it is of course for polygons having more than four sides. This conclusion stands in sharp distinction with that for the triangle. The reason for the difference lies in the fact that only for the triangle does equality of two sides ensure symmetry.

14. THE 'GOLDEN RECTANGLE' AND OTHERS

The so-called Golden Section of a linear segment is that which divides it in two segments in such a way that the longer segment is the mean proportional between the shorter segment and the whole segment.

The mathematician Luca Paciolo had claimed long ago* the central aesthetic importance of the proportion of the Golden Section. Within the last seventy-five years particularly this doctrine has been the subject of further speculation and of interesting experimental investigations. In particular the 'Golden Rectangle' (as we shall call it) whose sides are in the ratio of the Golden Section has attracted especial attention. This special rectangle with a ratio of 1.618 . . . , and so very nearly 8 to 5, is obviously agreeable to the eye. Furthermore it has the very interesting geometric property that if a square on the shorter side be removed, a smaller Golden Rectangle remains. Is it not then perhaps true that

* *De divina proportione*, Venice (1509).

AESTHETIC MEASURE

there resides in the Golden Rectangle some occult beauty which makes it superior to all other rectangular forms?

The psychologist Fechner * conducted well known experiments to ascertain if possible the most satisfactory rectangular shape, inclusive of the square. His results may be briefly summarized as follows: The square and, more especially, rectangles having dimensions approximating those of the Golden Rectangle, were generally considered to be the best.

However, Fechner used rectangular picture frames and a variety of other rectangular objects in his experiments. Such a frame is determined in its dimensions by the nature of the picture. In other cases also the dimensions are determined by similar considerations. To this extent his

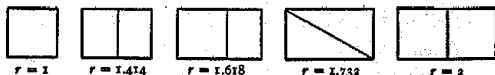


FIGURE 7

observations are irrelevant to the question of rectangular forms considered in isolation.

In thinking of the Golden Rectangle, it is well to keep in mind the special rectangle favored by Plato, made of the two halves of an equilateral triangle. For it, the characteristic ratio is 1.732. . . . It is also well to keep in mind the rectangle with ratio 1.414 . . . , which may be divided in two equal rectangles of the same shape as the original rectangle by a line parallel to the two shorter sides. Here the ratio is nearly 7 to 5. Furthermore the rectangle made up of two squares, with ratio 2 to 1 is to be noted.

Thus we find five rectangles (we include the square) with simple geometric properties. These are represented above, and the ratio r of the longer side to the shorter is given in each case. We conclude then that the Golden Rectangle is in no wise different from others in the respect that it possesses a simple geometrical property.

15. COMPARISON OF RECTANGULAR FORMS

If now we regard these and other rectangular forms as embodied in isolated vertical tiles, it will be found in the first place that the square tile

* See his *Vorlesungen über Psychologie*, Leipzig (1876), in particular chapter 19.

POLYGONAL FORMS

is the most effective. This fact agrees with the observation that the square is more frequently used in this manner than any other single rectangular shape. It will also be found that other rectangular forms are highly pleasing and not much to be distinguished from one another in aesthetic quality excepting as follows: a rectangular form nearly but not quite a square is disagreeable because of the effect of ambiguity (Chapter I, section 11); a rectangular form in which the ratio of the longer to the shorter side is too great suggests the segment of a line, and this effect of ambiguity is also disagreeable.

Moreover a rectangular form with a ratio as much as 2 to 1 is not well adapted to fill the circular field of effective vision. In consequence, such forms are not suitable in many cases, as for instance that of picture frames. But for a rectangular tile, such as we are considering, this is certainly not the case, since tiles with a considerably greater ratio are very pleasing and are used frequently for decorative purposes.

To avoid all suspicion of either ambiguity or lack of utility, we must therefore restrict ourselves further to rectangles whose characteristic ratio is plainly between 1 and 2. In consequence if we desire to choose rectangular forms which completely avoid undesirable factors, we are inevitably led to forms not very far from that of the Golden Rectangle, but among which are others like that with ratio $r = 1.414 \dots$ in the figure above. All such rectangular forms are both pleasing and useful.

These conclusions are in substantial agreement with the general theory of the present chapter, which accords a leading position to the square ($M = 1.50$) and all unambiguous rectangular forms ($M = 1.25$), but does not take account of the usefulness of rectangular forms. It is to be observed that usefulness corresponds to a connotative factor entirely outside of the scope of the theory.

Lipps has expressed himself to much the same effect as follows:*

It may now be looked upon as generally conceded . . . that the ratio of the Golden Section, generally and in this case [of the Golden Rectangle], is entirely without aesthetic significance in itself, and that the presence of this numerical ratio is nowhere the basis of any pleasant quality. . . .

In this way the question arises as to whence comes the indubitable special agreeableness of rectangles approximating that of the Golden Section. . . .

* *Ästhetik*, vol. 1, pp. 66-67, my translation.

AESTHETIC MEASURE

The rectangles in question are just those in which the smaller dimension is decisively subordinated to the greater. . . .

It has already been indicated above why the rectangle which approaches the square too closely pleases little. We term it awkward because of its ambiguity. On the other hand the rectangle in which one dimension falls too much behind the other . . . seems insufficient, thin, attenuated.

The experimental results of Fechner may also have been influenced by the fact that numerous persons, through their acquaintance with and liking for Greek art, or otherwise, have come to individualize and identify the Golden Rectangle. For them a connotative association of purely accidental character would be established in its favor. If a number of his experimental subjects were of this sort, Fechner's experimental results are easily explained.

16. USE OF RECTANGULAR FORMS IN COMPOSITION

Up to this point we have considered rectangular forms in isolation. It is interesting to note some facts concerning their use in composition, which undoubtedly have some residual effect upon our general appreciation of them even in isolation.

For use in composition it is very important that the rectangles have an infinitude of shapes, dependent on the arbitrary ratio of the sides, whereas the square has a single definite shape. Hence the rectangles provide a much more flexible instrument than does the square. It is, for example, obvious that the square shape is not in general suitable as a frame for a picture. This superior usefulness of the rectangle may establish in the long run a positive connotative factor in its favor.

Furthermore, in many of its uses, such as that of a picture frame, any obvious numerical ratio of dimensions such as 1 to 1 or 2 to 1 is to be avoided because it is often desirable that the rectangle be a purely neutral accessory, not producing irrelevant associations.

Finally it is to be observed that although the special forms of rectangles, like those mentioned above, have no especial significance when used in isolation, this is no longer true when they appear as elements in composition. For instance, the arrangement of two adjoining rectangular windows with $r = 1.414$. . . so as to form a single rectangle of the same shape might be decidedly pleasing architecturally, because the same shape is

POLYGONAL FORMS

discovered in an unexpected aspect. Similarly, three adjoining windows, the central one being a square and the outer ones equal Golden Rectangles with the shorter sides horizontal, might prove very pleasing.

17. THE FORMS OF FIVE- AND SIX-SIDED POLYGONS

Our survey of triangles and quadrilaterals has brought to light a number of the essential aesthetic factors which operate in more general cases. It would be tedious to continue with our analysis in full detail. If we did so, the facts for five- and six-sided polygons would be found to be on the whole similar to those already noted. We shall be content, therefore, to mention those factors which are not illustrated by the polygons of three or

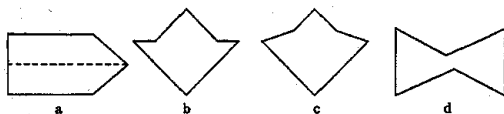


FIGURE 8

four sides, and then to pass on to such as are illustrated only by still more complicated polygons.

To begin with, let us recall that symmetry about an inclined axis has been observed to be of little or no aesthetic significance by itself. It is true that when there is symmetry about the vertical axis also, the matter is not so clear, but in that event there will be rotational symmetry as well. In consequence such symmetry about an inclined axis is to be looked upon as arising from the combined vertical symmetry and rotational symmetry, and so as logically dependent upon them. Hence symmetry about an inclined axis need not be considered in this case as a separate aesthetic factor.

What is the importance of symmetry about a horizontal axis when there is no vertical symmetry? This case is illustrated most simply by the pentagonal polygon (a) in Figure 8. In the first place it is clear that the symmetry about the horizontal axis is much more easily appraised by the eye than in any other direction except the vertical. Notwithstanding this fact, however, the symmetry about the horizontal axis is not enjoyed.

AESTHETIC MEASURE

A second factor already briefly alluded to is effectively isolated by a comparison of the two re-entrant hexagonal polygons (b) and (c). These are both of the same general type, but only in the first case (b) do two of the four re-entrant sides lie in a straight line and so support one another. It is obvious that (b) is notably superior to (c) just on this account. In general, then, we may expect unsupported re-entrant sides to operate unfavorably and so to correspond to a factor of negative type.

The psychological explanation of this situation appears to be as follows: in general the association of re-entrance is not a pleasant one; but if, when the eye follows a re-entrant side, another side is discovered in the same straight line there is a compensating feeling of satisfaction.

A new form of ambiguity is illustrated by the hexagon (d), in which two parallel sides are found nearly in the same straight line.

18. MORE COMPLICATED FORMS

The 90 polygons listed in the opposite Plates II-VII in order of decreasing aesthetic measure present graphically some of the principal types of polygons. Examination of these polygons yields a few further aesthetic factors of importance.

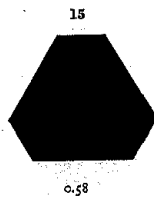
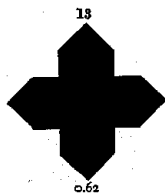
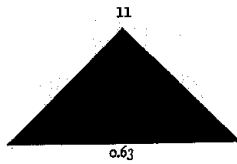
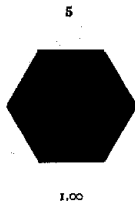
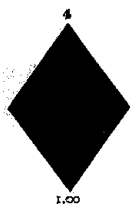
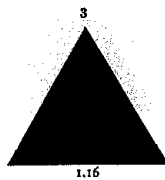
In the first place there is the obvious increasing complexity itself, which, beyond a certain point, is found to be tiresome. Furthermore it is evident that the convex polygon with a large number of sides is more likely to be pleasing than the re-entrant one, particularly if the latter contains a diversity of niches.

By a 'niche' of a polygon is meant any outer area lying within the minimum enclosing convex polygon.

Another important factor becomes obvious when the polygon is closely related to some uniform network of horizontal and vertical lines. The relationship may be direct, as in the case of the Greek cross (see (a) in the following figure); or it may be indirect, in that the polygon is directly related to a uniform diamond network with its sides equally inclined to the vertical (see (b)), while this network in turn suggests a uniform horizontal-vertical network.

Evidently the aesthetic factor of close relationship to a uniform horizontal-vertical or diamond network enhances greatly the aesthetic value

PLATE II

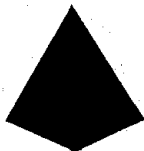


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17



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18



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20



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21



0.50

22



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24



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25



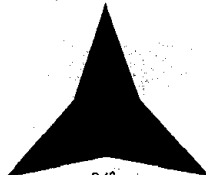
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27



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28



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29



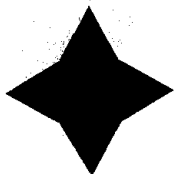
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33



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35



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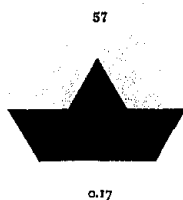
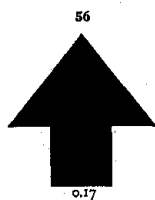
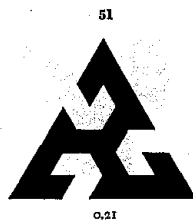
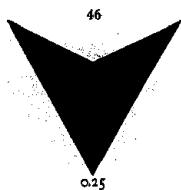


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45



0.29



61



0.14

62



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63



0.12

64



0.12

65



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66



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68



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69



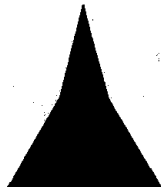
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PLATE VII

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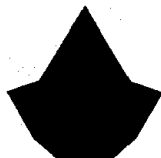
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85



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86



0.00

87



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88



-0.10

89



-0.11

90



-0.17

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of many of the polygons listed, e.g. the square (No. 1), the rectangle (No. 2), the diamond (No. 4), the six-pointed star (No. 6), the Greek cross (No. 9), the swastika (No. 41), etc. In the case of the square, rectangle, and diamond, the validity of this association may seem debatable, but so constantly do we observe all these forms in a uniform network that they seem always to suggest such a network; however, direct relation to a diamond network possesses less value than direct relation to a horizontal-vertical network. The associational basis of this factor in everyday experience is obvious: Systems of lines placed in the regular

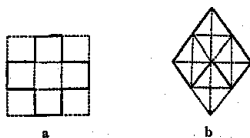


FIGURE 9

array of a uniform network are constantly met with, and their relationship to one another is intuitively appreciated.

The examination of more complicated polygons shows also that some kind of symmetry is required if their form is to be at all attractive. When this requirement is not met, relationship to a horizontal-vertical network, for instance, will not offset the deficiency. Evidently a further actual aesthetic factor in many cases is some accidental connotation, such as is present in the case of crosses and the swastika. Our theory leaves such connotative factors out of account.

19. ON THE STRUCTURE OF THE AESTHETIC FORMULA

According to the general theory proposed in the first chapter, we seek an aesthetic formula of the type $M = O/C$ where M is the aesthetic measure, O is the order, and C is the complexity. In the case of polygonal form, O will be separated into five elements of order:

$$O = V + E + R + HV - F.$$

The aesthetic factors encountered above are correlated in the following way with C and the five elements of order which make up O :

AESTHETIC MEASURE

C: complexity (-),
V: vertical symmetry (+),
E: equilibrium (+),
R: rotational symmetry (+),

HV: relation to a horizontal-vertical network (+),

F: unsatisfactory form (-) involving some of the following factors: too small distances from vertices to vertices or to sides, or between parallel sides; angles too near 0° or 180° ; other ambiguities; unsupported re-entrant sides; diversity of niches; diversity of directions; lack of symmetry.

It will be observed that the term *F* is an *omnium gatherum* for the negative aesthetic factors of unsatisfactory form. The various indifferent factors of type (*o*) play no part of course. Among these are equality of sides, and inclined or horizontal symmetry.

In the course of the technical evaluation of *C*, *V*, *E*, *R*, *HV*, *F*, and so of *M*, to which we now proceed, a simple mathematical concept, namely that of the group of motions of the given polygon, will be introduced (section 23). This concept will prove to be a useful adjunct.

20. THE COMPLEXITY *C*

The complexity *C* of a polygon will be defined as the number of indefinitely extended straight lines which contain all the sides of the polygon. Thus for a quadrilateral the complexity is evidently 4; for the Greek cross the complexity is 8, although the number of sides in the ordinary sense is 12; for the pinwheel figure shown in No. 53 the complexity is 10.

The psychological reasonableness of this empirical rule is evident: For convex polygons, and also for re-entrant polygons without any two sides in the same straight line, the complexity *C* is given by the number of sides. As the eye follows the contour of the polygon, the effort involved is proportional to this number. On the other hand, if there are several sides in one and the same straight line, the eye follows these in one motion. For example, in the case of the Greek or Roman cross, the eye might regard it as made up of two rectangles. These considerations suggest that the definition chosen for the complexity *C* is appropriate.

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21. THE ELEMENT V OF VERTICAL SYMMETRY

The agreeable organization of the polygon which results from vertical symmetry is obvious to the eye. By long practice we have become accustomed to estimating symmetry of this sort immediately. The corresponding element of order V is particularly significant.

We shall give to V the value 1 when the polygon possesses symmetry about the vertical axis, and the value 0 in the contrary case.

In other words, the element V will be a unit element of order, and, since there exist various polygons of pleasing quality, such as the swastika, which do not have vertical symmetry, we shall assign the value 0 rather than a negative value to V when there is no such symmetry.

When there is vertical symmetry, a center of repose for the eye will be found upon the axis of symmetry. Furthermore the polygon possessing such symmetry is felt to be in equilibrium. A large proportion of the 90 polygons listed possess vertical symmetry, and it can be verified that such symmetry is favorably felt.

22. THE ELEMENT E OF EQUILIBRIUM

Let us consider the second element E of order, concerned with equilibrium. It has been previously observed that when the polygon has vertical symmetry or rests upon a sufficiently extended horizontal base, it is felt to be in complete equilibrium.

In order to specify the requirements for complete equilibrium, we may note first that it is optical equilibrium which is referred to rather than ordinary mechanical equilibrium. For example, the pinwheel polygon No. 53 is in (unstable) mechanical equilibrium, inasmuch as the center of area lies directly above the lowest point. Nevertheless it does not give the impression of optical equilibrium. In the case when the polygon is not symmetric about the vertical, the feeling of optical equilibrium is only induced if there is a horizontal base with the extreme points of support far enough apart so that the center of gravity lies well between the vertical lines through these extreme points.

We shall agree that there is complete optical equilibrium if the center of area lies not only between these two vertical lines, but at a distance from

either of them exceeding one sixth that of the total horizontal breadth of the polygon. When this arbitrary condition is satisfied, as well as in the case of vertical symmetry, we shall give E the value 1. If the polygon does not satisfy this condition but is in equilibrium in the ordinary mechanical sense, we shall take E to be 0. Otherwise we shall take E to be -1 , inasmuch as the lack of equilibrium is then definitely objectionable.

In all of the listed polygons the selected orientation gives at least mechanical equilibrium, although in one case, No. 85, an equally favorable orientation exists lacking equilibrium ($E = -1$), in which the sides make an angle of 45° with the vertical.

23. THE GROUP OF MOTIONS OF A POLYGON

In the case when the polygon possesses rotational symmetry, it has been observed (section 3) that there is a least angle of rotation $360^\circ/q$. If the corresponding rotation be effected, the polygon will be returned as a whole to its initial position. If it be repeated q times, a complete revolution will be effected, so that every point returns to where it was at the outset.

There is a certain fundamental similarity between such rotational symmetry and the symmetry about an axis. In order to make this clear, let us recall that if a polygon be rotated through 180° about an axis of symmetry, it will return to its initial position. Thus the test for both kinds of symmetry is that a certain rotation restores the polygon as a whole to its initial position. In the case of rotational symmetry, the axis of rotation is perpendicular to the plane of the polygon at its center of area; while in the case of axial symmetry the axis of rotation is the axis of symmetry, and the rotation is a half revolution.

The collection of all these motions of rotation leaving a given polygon in the same position constitutes the 'group of motions' of the polygon. For reasons of convenience it is desirable to admit, as a conventional motion of rotation, the rotation about an arbitrary axis through an angle of 0° , which moves no point.

If A denotes one such rotation, and B the same rotation or any other, then the combination of the rotation A with the subsequent rotation B may be denoted by AB , and returns the polygon to its initial position.

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Hence the compound operation thus effected must be equivalent to a single rotation C of the group; that is, we may write, in brief mathematical symbolism, $AB = C$.

As a simple example let us consider a square. The 'group of motions' contains the following eight rotations: four rotations of 0° , 90° , 180° , 270° in its plane about the center of area; four rotations of 180° about the two diagonals and their two bisectors.

Suppose now that we follow the motion of rotation about one diagonal by a rotation about the other. The resultant effect is to transfer each point to the centrally symmetric point, just as if the polygon had been rotated in its plane through 180° . In other words the compound rotation formed from these two rotations of the group is equivalent to another rotation of the group. This fact illustrates the fundamental principle embodied in the symbolic equation $AB = C$.

The figures in the plane of the given polygon which arise from one such figure when all possible rotations of the group are applied to it will be said to be 'of the same type' as the given figure. In more mathematical terms all such figures are 'conjugate' under the given group.

For example, in the case of the square the four vertices, and also the four sides, are of the same type; vertical and horizontal directions are of the same type; the diagonals are of the same type. In the case of the rectangle all four vertices but only pairs of opposite sides are of the same type; similarly, vertical and horizontal directions are not of the same type, although the diagonals are.

Figures 'of the same type' are merely *corresponding* figures in the intuitive sense of the term.

The groups of motions of polygonal forms are of three possible species: (1) the groups of the regular polygons of q sides,* in which there is both axial and rotational symmetry; (2) the group of the isosceles triangle, in which there is only symmetry with respect to a single axis; (3) groups like those of the parallelogram ($q = 2$) and the swastika ($q = 4$) in which there is rotational symmetry with an angle of rotation $360^\circ/q$ but no symmetry about an axis.

* Inclusive of the case of a 'regular polygon of two sides' formed by a single line. The group in this case is the same as that of a rectangle.

24. THE ELEMENT R OF ROTATIONAL SYMMETRY

In dealing with the element R of rotational symmetry, we are guided by certain obvious visual facts. The simplest type of rotational symmetry is that of central symmetry in which the least angle of rotation is 180° . The parallelogram illustrates this possibility. It is clear that central symmetry is at once appreciated by the eye.

In the case of central symmetry any line through the center of area divides the polygon into two figures 'of the same type.' For example, a diagonal of the parallelogram divides it into two triangles of the same type. On this account any half of such a polygon determines the other half. Likewise in the case of axial symmetry, the half of the polygon on one side of the axis determines the half on the other side, and these two halves are also of the same type. In other words, central symmetry requires the same extent of organization within the polygon as does axial symmetry.

For this reason we take the element R to be 1 in the case when there is only central symmetry, just as we take V to be 1 in the case of axial (vertical) symmetry.

Let us turn next to the case where the group of motions is that of a regular polygon. Obviously the only effective orientations are those in which an axis of symmetry lies along the vertical direction; the rotational symmetry is appreciated more for larger values of q .

Here the polygon can be broken up into q partial symmetric polygons situated symmetrically around the center of area. By analogy with the case of central symmetry, it is therefore natural to assume that R varies in proportion to q ; for, any one of these q component parts determines all of the others. Since when q is 2, R is 1, we are in general led to define R as $q/2$.

However, the element of rotational symmetry is only effective up to a certain point, after which there is no further increase. When q is 6 or exceeds 6, the circle circumscribed about the polygon is very clearly suggested and the impression of rotational symmetry becomes complete. For these reasons, in the case of vertical and rotational symmetry combined we define the element R as $q/2$ for q not greater than 6, and as 3 for larger values of q .

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There is a re-entrant case in which the element R is felt almost equally favorably, despite a lack of vertical symmetry, namely when the minimum convex polygon enclosing the given polygon is symmetric about a vertical axis, and none of its vertices abut on the niches. Polygons Nos. 41, 51, and 69 illustrate this case; here the enclosing convex polygon, with its q axes of symmetry, is so strongly outlined as to suggest vividly the rotational element, and we define R as in the preceding case.

On the other hand, even though the minimum enclosing convex polygon is symmetric about a vertical axis, the same effect is not felt if its vertices abut on the niches of the given polygon. This possibility is illustrated by polygons Nos. 53, 67, 84, 88, and 90. In explanation of this difference in effect, it may be observed that the axes of symmetry of the enclosing polygon are hardly felt as such under these circumstances.

In such cases and in others when the enclosing polygon is not axially symmetric, there is central symmetry when q is even. Such symmetry is appreciated immediately, though the rotational symmetry as such plays a negligible rôle. Accordingly we take R to be equal to 1 here. Polygons Nos. 48, 53, and 67 illustrate this possibility.

On the other hand, when q is odd the effect is less favorable still. Nos. 79 and 88, 89, 90 illustrate this situation for convex and re-entrant polygons respectively. Even in the convex type, most persons will scarcely be aware of the rotational symmetry or will find it to be disagreeable. Thus in this last case, as well as in any case when there is no rotational symmetry, we are led to take R to be 0.

25. THE ELEMENT HV OF RELATION TO A HORIZONTAL-VERTICAL NETWORK

As has previously been noted, in many polygons of the list there is evidently a close relationship of the given polygon to a uniform horizontal-vertical network, and this relationship is decidedly pleasing.

The corresponding element HV in O is connected with certain motions of the plane in much the same way as the element V is connected with a motion of rotation about a vertical axis, and the element R with a motion of rotation about a center. In fact such a uniform network evidently returns as a whole to its initial position, when certain translatory motions

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of the plane are made. Such motions will take the polygon to a new position related to the same network. In an incomplete way, then, the element HV is connected with motions of the plane just as the elements V and R are.

The most favorable case is that in which the polygon has all of its sides lying along the lines of a uniform network of horizontal and vertical lines, in such wise that these lines completely fill out a rectangular portion of the network. In this case only do we take HV to be 2. Nos. 1, 2, 9, 23, 25, 26, 29, 41, 44, and 55 illustrate this possibility.

The choice of 2 as the corresponding index of HV is suggested by the fact that there are essentially *two* kinds of independent translatory motions which return the network to its original position, namely a translation to the right or left, and a translation up or down. Any other translation may be regarded as derivable by combination from these two alone.

Another similar case is that in which the sides of the polygon all lie upon the lines of a uniform network formed by two sets of parallel lines equally inclined to the vertical, and fill out a diamond-shaped portion of the network (see Nos. 4, 36, and 45). But the effect here is less favorable, so that we take HV to be 1.

It becomes necessary at this stage to assign an arbitrary index in all cases. From the purely geometrical point of view, the degree of coincidence of a polygon with such a horizontal-vertical or diamond network may range from the case of maximum coincidence, as specified above, to the case of practically no coincidence, through a series of intermediate degrees. It is thus suggested that a corresponding graded index may be required. It is found, however, that as soon as there is a slight deviation from complete coincidence, the pleasantness of the effect diminishes markedly, and for further deviation vanishes entirely.

Hence we shall select the following empirical rule: HV is to be 1 if the polygon fills out a rectangular portion of a horizontal-vertical network, save for the following exceptions: one line of the polygon and the other lines of the same type (see section 23) may fall along diagonals of the rectangular portion or of adjoining rectangles of the network; one vertical and one horizontal line of this portion, as well as other lines of the same type, may not be occupied by a side of the polygon.

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Illustrations of this case $HV = 1$ are furnished by the polygons Nos. 13, 42, 43, 49, 50, 53, 62, 66, and 78 of the list.

The element HV will also be defined to be 1 if the polygon fills out a diamond-shaped portion of a diamond network save for entirely analogous exceptions: one line of the polygon and the other lines of the same type may fall along diagonals of the diamond-shaped portion or of adjoining diamonds of the network; one line of this portion as well as the other lines of the same type may not be occupied by a side of the polygon. The polygons Nos. 5, 6, 24, and 68 are illustrative of this case.

Moreover when HV is 1 we shall demand that at least two lines of either set of the network are occupied by a side.

In all other cases whatsoever we shall take HV to be 0.

It is obvious that the above determination of indices for the element HV is largely arbitrary. Nevertheless it seems to correspond to the facts observed.

26. THE ELEMENT F OF UNSATISFACTORY FORM

There remains to be treated the negative constituent F in O , which we have described as an *omnium gatherum* of the negative elements of order (section 19).

The case in which F is 0 corresponds to satisfactory form. Here the analysis made in the earlier sections suggests the following conditions: (1) the minimum distance from any vertex to any other vertex or side, or between parallel sides, is not to be too small — for definiteness we shall demand that it be not less than one tenth the maximum distance between points of the polygon; (2) the angle between two non-parallel sides is not to be too small — for definiteness let us say not less than 20° ; (3) more generally, all other ambiguities of form are to be avoided — for definiteness let us demand that no shift of the vertices by less than one tenth their distance to the nearest vertex can introduce a further element of order in V , R , or HV ; (4) there is to be no unsupported re-entrant side; (5) there is to be at most one type of niche; (6) there are to be at most two types of directions, provided that vertical and horizontal directions (when both occur) are counted together as one; (7) there is symmetry to the extent that V and R are not both 0.

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The first three of these requirements eliminate ambiguity of form. Of them the first two deal with kinds of ambiguity explicitly mentioned above. The third requirement takes account of the kind of ambiguity which is found to arise, for example, when a triangle is nearly but not quite isosceles or equilateral.

The other requirements are also suggested by the analysis given above. In particular the fifth and sixth requirements are based on the fact that while one type of niche is compatible with satisfactory form, as in the Greek cross No. 9, and two types of directions are not excessive (provided vertical and horizontal directions are counted together as one), it is not possible to go further without impairment of satisfactory form.

If one and only one of the above conditions fails and that to a minimum extent (e.g. there is *one* type of unsupported re-entrant side), we take F to be 1. In all cases where there is more than a single violation of these conditions we take F to be 2.

For all of the polygons Nos. 1-22 inclusive, F is 0. No. 23 is the first polygon for which F is 1 because of one type of unsupported re-entrant side; No. 24 is the first polygon for which F is 1 because of two types of niches. The earliest polygon for which F is 1 because of diversity of directions is No. 60. The earliest for which F is 2 is No. 55.

27. RECAPITULATION OF DEFINITION OF AESTHETIC MEASURE

For purposes of convenient reference let us state concisely the above definition of the aesthetic measure of a polygon in vertical position:

The formula is

$$M = \frac{O}{C} = \frac{V + E + R + HV - F}{C}$$

with the following definitions:

C

C is the number of distinct straight lines containing at least one side of the polygon.

V

V is 1 or 0 according as the polygon is or is not symmetric about a vertical axis.

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E

E is 1 whenever *V* is 1.

E is also 1 if the center of area *K* is situated directly above a point *D* within a horizontal line segment *AB* supporting the polygon from below in such wise that the lengths *AD* and *BD* are both more than 1/6 of the total horizontal breadth of the polygon.

E is 0 in any other case when *K* lies above a point of *AB*, even if *A* and *B* coincide.

E is -1 in the remaining cases.

R

R is the smaller of the numbers $q/2$ and 3 in the case of rotational symmetry, provided that the polygon has vertical symmetry or else that the minimum enclosing convex polygon has vertical symmetry and that the niches of the given polygon do not abut on the vertices of the enclosing polygon.

R is 1 in any other case when *q* is even (i.e. if there is central symmetry).

R is 0 in the remaining cases.

HV

HV is 2 only when the sides of the polygon lie upon the lines of a uniform horizontal-vertical network, and occupy all the lines of a rectangular portion of the network.

HV is 1 if these conditions are satisfied, with one or both of the following exceptions: one line and the others of this type may fall along diagonals of the rectangular portion or of adjoining rectangles of the network; one vertical line and one horizontal line of the portion, and the others of the same type, may not be occupied by a side. At least two vertical and two horizontal lines must be filled by the sides however.

HV is also 1 when the sides of the polygon lie upon the lines of a uniform network of two sets of parallel lines equally inclined to the vertical, and occupy all the lines of a diamond-shaped portion of the network, with the following possible exceptions: at most one line and the others of the same type may fall along diagonals of the diamond-shaped portion or of adjoining diamonds of the network; one line of the diamond-shaped

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portion and the others of its type may not be occupied by a side. At least two lines of either set of parallel lines in the network must, however, be occupied by the sides.

HV is 0 in all other cases.

F

F is 0 if the following conditions are satisfied: the minimum distance from any vertex to any other vertex or side or between parallel sides is at least $1/10$ the maximum distance between points of the polygon; the angle between two non-parallel sides is not less than 20° ; no shift of the vertices by less than $1/10$ of the distance to the nearest vertex can introduce a new element of order V , R , or HV ; there is no unsupported re-entrant side; there is at most one type of niche and two types of directions, provided that vertical and horizontal directions are counted together as one; V and R are not both 0.

F is 1 if these conditions are fulfilled with one exception and one only.

F is 2 in all other cases.

28. APPLICATION TO 90 POLYGONS

The 90 polygons arranged in Plates II-VII in order of decreasing aesthetic measure according to the formula furnish in themselves a severe test of its approximate accuracy. If, upon scanning these polygons from the first to the last, the reader feels a gradual diminution in aesthetic quality,* the underlying theory may be regarded as justified. The following facts should be observed.

Many polygons have important connotative elements of order which will have an effect upon the aesthetic judgment, unless one abstracts from them explicitly. Thus the stars Nos. 6, 8, 40, the different crosses Nos. 9, 13, 29, 49, and the swastika No. 41, have their aesthetic value definitely enhanced by the corresponding connotations. The importance of the positive connotative elements of order in these cases becomes obvious if one inverts the Roman cross No. 49, and observes how its aesthetic effect is thereby impaired.

Likewise certain other polygons have important negative connotative elements of order. Thus polygons Nos. 23, 38, and 39 strongly suggest

* Those with the same aesthetic measure are to be grouped together of course.

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the respective letters H, I, and X; this association operates to diminish their aesthetic value. Similarly the association of No. 37 with the outline of a rectangular box is displeasing. The polygon No. 56, of a shape which suggests either a pine tree or an arrowhead, will be increased or diminished in value perhaps, according as the first or second of these associations is uppermost.

Other connotations of a geometrical type ought to be noted. Thus the swastika, No. 41, suggests strongly two broken lines. A polygon of many small sides inscribed in a regular curve (see Nos. 10, 12, and 16) will suggest the curve so strongly as to exclude the consideration of the polygon as such.

In testing the validity of the formula as applied to such a set of polygons, all these connotations must be borne in mind.

Secondly, it is to be recalled that no special attempt is made here to classify indifferent polygons. Thus, 18 of the last 21 polygons are of measure 0, and the last 3 polygons are of negative measure. Consequently there is no distinction between the first 18 of these, and perhaps it is as well not to take too seriously the indication that the last 3 polygons are definitely worse than the others.

In the third place the formula is not a highly sensitive one, since the change produced by any single element of order is considerable. For example, all triangles whatsoever have an aesthetic measure of $7/6$, $2/3$, 0, $-1/3$, $-2/3$, or -1 according to the definition. No other gradations are possible.

If then, after laying aside connotations as far as possible, there is felt to be a gradual diminution in aesthetic value as the polygons are looked at in succession, so that polygons whose aesthetic measures are substantially unequal are properly arranged, while those of almost equal measures are of nearly equal attractiveness, the formula must be regarded as justified. We may conjecture in this event that the general theory is correct in its essential features. Of course considerable variations in individual judgments are to be expected.

In classes at Columbia University (summer, 1929) and Harvard University (summer, 1930) I obtained the consensus of aesthetic judgment as to the arrangement of these polygons. The results so obtained were

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found to be in substantial agreement with the arrangement obtained by the formula.

29. POSSIBLE MODIFICATIONS

It is interesting to ask whether the appreciation of polygonal form has undergone a process of gradual evolution in the past, and whether further modifications are to be looked for.

The appreciation of symmetric form — as manifested by the human figure, by the sun, moon, etc. — and of equilibrium must have been present from the earliest times. The utilization of rectangular networks must go back at least as far as primitive architectural design. Hence it seems likely that the enjoyment of polygonal forms has not changed very much in character since the dawn of civilization.

If, for any reason, polygonal forms were to receive much attention, it seems certain that aesthetic appreciation of them would undergo further interesting development. We shall merely point to two obvious possibilities. From a mathematical point of view other uniform networks, such as that of equilateral triangles, have just as much geometric interest as the rectangular ones. As soon as the 'normal observer' becomes familiar with polygons closely related to networks of these new types, he will find that these polygons gain in aesthetic value. Polygons such as Nos. 51, 69, and 89 are of this kind. If such a development were to take place, the definition of the element *HV* of relation to a uniform network would require modification. Moreover, if polygonal forms came into greater use, more elaborate forms would become attractive. For example, perspective correct representations of polygons found in the above list would be appreciated as such. This too would necessitate appropriate further modification in the definition of aesthetic measure.

30. THE MATHEMATICAL TREATMENT OF AESTHETIC QUESTIONS

A complete theory such as that which precedes can be used as a logical tool in order to answer aesthetic questions by purely mathematical (logical) reasoning. For example, let us propose the following question: Which is the most beautiful of all polygonal forms?

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To answer this question we observe first that the positive elements V , E , R , and HV cannot exceed 1, 1, 3, and 2 respectively in numerical value, while the most advantageous value of F is 0. In consequence O can never exceed 7, whence it appears that the aesthetic measure M of any polygon of complexity C cannot exceed $7/C$.

It is known that, for the square with one side horizontal, O is 6 and C is 4, so that M is 1.50. For any polygon for which C is as great as 5, it is clear that M will not exceed $7/5 = 1.40$. Hence we can conclude that any conceivable polygon for which M is as great as for the square has a complexity of 3 or 4.

But if the complexity C is 3, all the sides of the polygon lie on three straight lines. The only polygons of this kind are the triangles. However, for the scalene and isosceles triangles R and HV are 0, so that O is at most 2, and M is at most $2/3 = .67$. Hence these triangles can be excluded from consideration. For the equilateral triangle M is immediately found to be only $7/6 = 1.16 \dots$, so that it can be excluded as well.

It only remains to consider the possibility that the complexity C is 4, in which case the polygon lies wholly upon four straight lines. Such a quadrilateral cannot have network value ($HV = 2$ or 1) unless these four lines are two pairs of parallel lines; in this event it must be a square ($M = 1.50$), rectangle ($M = 1.25$), or diamond ($M = 1.00$). In any other case HV and either V or R are 0, and it is clear that O is at most 2 and M at most $2/4 = .50$.

It follows then as a 'theorem' that the square with horizontal sides with $M = 1.50$ is the best of all possible polygonal forms. Obviously such mathematical treatment upon the basis of the theory becomes a mere game if carried too far. It is only desirable to refer to this possibility of the theory in order to indicate its completeness.

31. ON UNCERTAINTY AND OPTICAL ILLUSIONS

In what precedes it has been assumed that the observer identifies the polygon completely from a mathematical point of view. Any consideration of the degree to which this assumption is actually valid would introduce difficult psychological questions.

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Sometimes the degree of uncertainty is increased by certain systematic optical illusions. It is a well known fact, for instance, that there is a definite tendency to overestimate vertical distances as compared to horizontal distances. Because of this illusion, a rectangle not a square may be adjudged to be a square. In all of what follows, illusions and uncertainties, whether visual or auditory, will not be taken into account.

CHAPTER VIII

THE MUSICAL QUALITY IN POETRY

I. THE TRIPARTITE NATURE OF VERSE

IN the analysis of poetry we shall make a division of the aesthetic factors into those pertaining to significance, to musical quality, and to metre. The significance of a poem is to be found in its 'poetic ideas', which must be expressed in accordance with Hemsterhuis's general dictum of the "greatest number of ideas in the shortest space of time." In other words the poet must begin with a poetic vision, and then by the use of poetic license, effective figures of speech, and onomatopoeic devices obtain an adequate embodiment of this vision in terse form; that is, in a form much shorter than would be necessary for an equally adequate expression in the language of prose.

One can find frequent appreciations of this first factor in poetry. The poet Shelley when he defined poetry as "the expression of the imagination" was alluding to the primacy of the poetic idea. The well known important rôle of poetic freedom as an essential element in expression is indicated by Pope as follows:

Thus Pegasus, a nearer way to take,
May boldly deviate from the common track.

The terseness of poetry is stressed by Voltaire: "Poetry says more and in fewer words than prose."

This factor of significance is evidently essentially connotative in its nature and beyond any possibility of formal analysis. It is perhaps the most important single element in poetry, and yet poetry without musical quality and metre is not properly poetry at all.

The other two factors of musical quality and metre are much more formal in their nature. It is because of these factors, which correspond to those of harmony and rhythm in music, that Fuller wrote "Poetry is music in words, and music is poetry in sound."

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We shall say nothing of metre in what follows except to mention its similarity to rhythm in music. This similarity has been considered by Lanier in his *Science of English Verse*, who goes so far as to use musical notation in order to specify metre. It is to be expected that metre in poetry will be susceptible of mathematical analysis in regard to aesthetic effect, just as is rhythm in music. As far as metre is concerned, the poet must first select an appropriate and sufficiently pliable metric form, and then in the inevitable deviations from its rigid execution be guided by a delicate sensibility of the effect upon the ear.

The remaining factor of musical quality, to which we devote our attention, is in certain respects the most characteristic one. Thus Butler wrote:

For rhyme the rudder is of verses
With which, like ships, they steer their courses.

And Shelley lays a similar emphasis: "The language of the poets has ever affected a sort of uniform and harmonious recurrence of sound, without which it were not poetry and which is scarcely less indispensable to the communication of its influence than the words themselves without reference to that peculiar order." It is here that the ingenuity of the poet is much more exercised than in following some more or less arbitrary metric form.

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It must be borne in mind that the notion of musical quality, as separable from significance and metre in a poem, is only approximately tenable. In such a line as the following of Tennyson's:

The league-long roller thundering on the reef,

there is expressed an imaginative theme for the eye and ear in onomatopoeic language of the utmost terseness, having appropriate metric structure and unusual musical quality. This line is, however, to be looked upon as produced by the intimate union of these factors, and so as being much more than a mere aggregation of them all. Nevertheless it is certain that the musical factor is to a large extent appreciated by itself, so that two poems can be intuitively compared in regard to their musical quality, almost regardless of their significance or metric form.

3. RHYME

The first and most obvious of the simple musical factors in poetry is rhyme. Here, in Western poetry, one group of sounds is compared with another in the following manner. The initial (elementary or composite) consonantal sound of the first group corresponds to a distinct initial consonantal sound of the second group; the remaining sounds of the two groups are identical. It is required furthermore that the two groups contain one accented vowel sound having the same relative position in the two groups, and that these groups terminate with a word. The following are simple instances of such rhyming groups: *Khan, ran; decree, sea; fire, desire; numbers, slumbers; remember, December*. There are also certain slight licenses that are admitted in rhyme, to which we can only allude in passing.

The element of rhyme in poetry is analogous to the element of melodic contrast in melody, and plays a considerable part.

4. ASSONANCE

Under certain circumstances the repetition of a vowel sound gives rise to a pleasurable feeling of assonance; it is only in this narrow sense of vowel repetition that we shall employ the word 'assonance.' The effect of assonance is increased by further repetitions, at least up to a certain point, after which the excess of assonance becomes unpleasantly monotonous and disagreeable.

The factor of assonance in poetry is analogous to that of repetition in melody. Its use is illustrated, for instance, by the following opening lines from Poe's poem, 'The Bells':

Hear the sledges with the bells —
Silver bells!

What a world of merriment their melody foretells!

These two lines contain the vowel sound of *e*, as in 'bells,' eight times in twenty-three syllables, and yet this repetition is not felt as excessive. Throughout Poe's poem the same sound is repeated extremely often. This repetition would be felt to be monotonous if the recurrence did not contain an onomatopoeic suggestion appropriate to its subject, 'The Bells.'

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5. ALLITERATION

By 'alliteration' we shall mean only such repetition of consonantal sounds as is felt by the ear. For instance, in the first two lines from 'The Bells' there is alliterative occurrence of the sound *s* = *z*.

A more nearly excessive use of alliteration and assonance occurs in the following interesting illustrative stanza given by Poe in his essay, 'The Rationale of Verse':

Virginal Lilian, rigidly, humbly dutiful;
Saintlily, lowlily,
Thrillingly, holly,
Beautiful

Here the consonant sound of *l* appears no less than sixteen times in the thirty syllables while the short vowel sound *i* occurs thirteen times. Observe also the five times occurring repeated syllable, *lil*. Of course a fundamental flaw in this stanza is the appearance in it of two non-existent adverbs, 'saintlily' and 'lowlily.'

Alliteration, like assonance, is analogous to the element of repetition in melody.

6. THE MUSICAL VOWEL SOUNDS

In general the vowel sounds are smoother than the consonantal sounds; and among the vowel sounds there are certain ones which are especially musical in quality, notably the *a* as in *art*, the *u* as in *tuneful*, *beauty*, and the *o* as in *ode*. When these appear sufficiently frequently, they impart their soft musical character to verse.

The mathematician Sylvester in his *Laws of Verse*, to which we shall refer subsequently, has a footnote of interest in this regard: "I can not resist the temptation of quoting here from a daily morning paper the following unconsciously chromatic passage . . . : 'The last portion of the shadow of the earth has been passed through by the moon which then again sailed in its full orb of glory through the dark blue depth.'" Besides possessing pleasant alliterative and assonantal elements, this sentence contains ten musical vowels as follows: *last*, *shadow*, *passed*, *through*, *moon*, *full*, *glory*, *through*, *dark*, *blue*. The aesthetic element introduced in poetry by these

musical vowels may be compared with that due to the primary chords in melody.

There are doubtless certain special instances where the play of vowels and consonants produces a kind of musical tune in a poem, analogous to melody. Lanier (*loc. cit.*) says in this connection: "Tune is . . . quite as essential a constituent of verse as of music; and the disposition to believe otherwise is due only to the complete unconsciousness with which we come to use these tunes . . . in all our daily intercourse by words."

However, it would be very difficult to disengage this element in precise form, and it is not certain how far its rôle is really independent of that of the musical sounds, alliteration, and assonance, already taken account of. For these reasons we do not attempt to deal separately with this somewhat obscure element of 'tune' in a poem.

7. 'ANASTOMOSIS'

Another aesthetic factor which is agreeable is that which results from the fact that a poem is easily spoken. In general this is brought about when the consonantal sounds are simple rather than composite, and not much more numerous than the more easily pronounced vowel sounds. Sylvester used the term 'anastomosis' to express this desirable quality (*loc. cit.*): "Anastomosis regards the junction of words, the laying of them duly alongside of one another (like drainage pipes set end to end, or the capillary terminations of the veins and arteries) so as to provide for the easy transmission and flow of the breath . . . from one into the other."

From our point of view this factor is not a unitary one, but needs to be split up into two others. On the one hand the composite consonantal sounds increase the complexity, *C*, of the poem, which, in accordance with our theory, is correlated with the effort required in speaking it. Such complexity is heightened also when a word ends with a consonantal sound and the following word begins with a sufficiently different consonantal sound. Hence such sequences are to be avoided as far as possible.

On the other hand, quite regardless of this factor of complexity, an excess of consonantal sounds is felt to be decidedly harsh and disagreeable. For example in the catch:

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Midst thickest mists and stiffest frosts,
With strongest fists and stoutest boasts,
He thrusts his fists against the posts,
And still insists he sees the ghosts —

there are approximately twenty-four consonantal sounds as against only eight vowel sounds in the first line, and a similar relationship holds in the other three lines. Here we have serious consonantal excess, a negative aesthetic factor.

Thus, from our point of view, 'anastomosis' is secured by avoidance of consonantal excess and by the diminution of 'complexity' as far as possible.

8. POE'S CONCEPT OF VERSE

We have now alluded to the principal elements of order involved in the musical quality of verse. While there exists, so far as I have discovered, no formulation but my own of its 'aesthetic measure,' nevertheless both Poe and Sylvester come within striking distance of an analysis of musical quality and metre in verse. The following quotations from Poe (*loc. cit.*) indicate sufficiently his point of view:

Verse originates in the human enjoyment of equality, fitness. To this enjoyment, also, all the moods of verse — rhythm, metre, stanza, rhyme, alliteration, the *refrain*, and other analogous effects — are to be referred.

The perception of pleasure in the equality of *sounds* is the principle of *Music*.

Anyone fond of mental experiment may satisfy himself, by trial, that, in listening to the lines, he does actually, (although with a seeming unconsciousness, on account of the rapid evolution of sensation,) recognize and instantly appreciate (more or less intensely as his ear is cultivated,) each and all of the equalizations detailed. The pleasure received, or receivable, has very much such progressive increase, and in very nearly such mathematical relations, as those which I have suggested in the case of the crystal.

Evidently in Poe's statements there is an assertion of the quantitative, additive character of the aesthetic pleasure produced, which corresponds closely with our concept of the order, *O*.

The notion of the complexity, *C*, does not enter explicitly in his analysis, but doubtless in the background of his ideas is the implicit requirement of a large *density* of such relations of 'equality' as Poe termed them. By equality Poe had in mind not only identity in sound but also metric equality, so that, in effect, he proposed to deal with both mathematical

factors of metre and musical quality. He refers nowhere to the factor of significance, although of course it is also implicit in the background of his ideas. It is interesting that, precisely in the respect of significance, Poe fails to be of the first rank among poets.

9. SYLVESTER'S CONCEPT OF VERSE

There is no doubt that Sylvester's concept of verse was much influenced by that of Poe. However, Sylvester went further than Poe in approaching the point of view demanded by our mathematical theory, as the following quotations show (*loc. cit.*):

In poetry we have sound, thought, and words . . . ; accordingly the subject falls naturally into three great divisions, the cogitative, the expressional, and the technical; to which we may give the respective names of Pneumatic, Linguistic, and Rhythmic. It is only with Rhythm that I profess to deal. This again branches off into three principal branches — Metric, Chromatic and Synectic.

I touch very briefly on this branch [of Metric] accepting, in regard to it, the doctrine of Edgar Poe. . . .

Metric is concerned with the discontinuous, Synectic with the continuous, aspect of the Art. Between the two lies Chromatic, which comprises the study of the qualities, affinities and colorific properties of sound. Into this part of the subject, except so far as occasionally glancing at its existence and referring to its effects, I do not profess to enter. My chief business is with Synectic.

This, also, on a slight examination, will be found to run into three channels — *Anastomosis*, *Symptosis*, and between them the main flood of *Phonetic Syzygy*.

Evidently Sylvester's 'Pneumatic' and 'Linguistic' fall under what we have termed the significance (with appropriate brief expression), while his 'Rhythmic' embraces both musical quality and metre. Furthermore he is concerned with musical quality rather than metre.

His last statement is the important one. As we have observed, his principle of anastomosis corresponds roughly to the requirement of as little complexity, *C*, as possible. On the other hand his principle of symptosis "deals with rhymes, assonances (including alliterations so-called), and clashes (this last comprising as well agreeable reiterations, or congruences, as unpleasant ones, i.e. jangles or jars)." It involves then the same elements of order as are classified above. Consequently symptosis is more or less the counterpart of our order, *O*.

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Thus his 'Phonetic Syzygy' — the effective combination of 'Anatomosis and 'Symptosis' — corresponds in a qualitative sense to our aesthetic measure of musical quality. It does not appear that Sylvester believed the aesthetic effect of poetry to be quantitatively measurable as Poe had conjectured it to be.

10. ON PHONETIC ANALYSIS

As a first step towards the formulation of an aesthetic measure of musical quality in poetry, it is necessary to introduce certain agreements, as simple as possible, concerning phonetic analysis. We shall recognize only the following vowel sounds as distinct: *aid, add, art; eve, ell; isle, ill; ode, or; use, lune, full, lull, urn; oil; out*. All vowel sounds distinct from these are to be assigned to the nearest one of these sounds.

The sounds of *a* as in *art*, *u* as in *tuneful*, *beauty*, *o* as in *ode*, will be called 'musical' because of their agreeable quality, similar to that of a pure musical note.

The elementary consonantal sounds are taken to be the following:

b, d, f, g, h, j, k, l, m, n(g), p, r, s, t(h), w(h), y, z.

The ordinary *g* sound is not regarded as present in *ng* although *n* is present. Likewise the aspirate *h* is not regarded as present in *th* and *wh* although *t* and *w* are present. Neither *c* nor *h* are regarded as present in *ch*, nor *h* in *sh*; all other composite consonant sounds will be analyzed in the manner indicated by the usual spelling. The phonetic justification of these simple conventions is obvious. Vowel and consonantal sounds not used in the English language need not be considered of course, since we are dealing with English poetry only.

11. THE COMPLEXITY *C*

The complexity *C* of any part of a poem — as, for instance, of a single line — is simply the total number of elementary sounds therein, increased by the number of word-junctures involving two adjacent consonantal sounds of the same line, which do not admit of liaison. The following pairs of distinct consonantal sounds will be taken to admit of liaison: *b, p; d, t; f, v; g, k; m, n; s, z*. Likewise if the second sound is an aspirate *h*, liaison will be admitted.

The justification for the definition is self-evident: We may with fair approximation to the facts consider each vowel sound and elementary consonantal sound as being equally difficult to pronounce. Furthermore, adjacent but distinct consonantal sounds belonging to different words are nearly as difficult to pronounce as if there were a vowel sound between them. The definition takes account of all these facts in a simple way.

12. THE ELEMENT *2r* OF RHYME

The number of sound groups in the line or part of the poem under consideration, which rhyme with at least (and in general only) one sound group earlier in the same line or in an earlier line will be designated by *r*. The corresponding element of rhyme will be taken to be *2r*. It is seen then that the index 2 is assigned to the tone of feeling due to a single rhyme. Later we shall assign an index of 1 to the single alliteration or assonance. The higher rating of the rhyme is justified by the greater intensity of the tone of feeling which each rhyme induces.

The reason for counting each rhyming group only once, even though it rhymes with more than one earlier group, is that each rhyming group is in general set against one particular contrasting earlier group. By general consent certain slight liberties are allowed in rhyming. For instance the *u* sounds as in *use* and *lune*, are allowed to replace one another. We shall not undertake to enumerate these exceptions.

The evaluation of the element *r* is immediate in all cases. For example in the case of Poe's stanza there are the following rhyming groups:

lowlily, holily; dutiful, beautiful.

Thus *r* is 2 and the element *2r* of rhyme is 4 in this case.

13. THE ELEMENT *aa* OF ALLITERATION AND ASSONANCE

In dealing with the element of alliteration and assonance, it is necessary first to specify limits within which the repetition of a consonant or vowel sound is pleasantly felt, and then to decide in how far repetitions may accumulate before there is alliterative or assonantal excess. The short nondescript vowel sound as in *the, attention*, which may be considered to be a short form of *u* as in *lull*, will not be regarded as assonantal under any circumstances.

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In order to give an empirical definition of the element *aa* of alliteration and assonance, we shall introduce certain technical terms. A 'leading sound' of a line will be defined as one which either is part of an accented syllable, or occurs as the initial sound of a line, or is part of a rhyming group. Such leading sounds are those which most impress the ear.

A sound, *a*, in a certain position will be said to be 'directly connected' with the same sound or sounds in other positions as follows: (1) the same sound is found again in the same word as *a* or in an adjoining word; (2) the same sound occurs as a leading sound earlier in the line than *a*, or in the last half of the preceding line provided *a* occurs in the first half of its line; (3) *a* follows a pair of the same sounds earlier in the line which are in the same word or in adjoining words; (4) the same sound occurs as a rhymed leading sound or as a leading sound in the same relative position as *a*, either in the preceding line or in an earlier line rhyming (or identical) with the line containing *a*; (5) the same sound occurs in an identical syllable earlier in the line than *a* or in the last half of the preceding line provided *a* occurs in the first half of its line.

If there is an odd number of feet in a line, the middle foot will be considered to be part of both the first and second halves of the line in applying these definitions; moreover, if a line contains only two feet, we shall include both feet in either half of the line, by special convention.

When a sound is repeated under these conditions, the repetition produces an effect of alliteration or assonance which is felt agreeably.

With these definitions in mind we shall define the element *aa* of alliteration and assonance in the line or group of lines under consideration as the number of sounds directly connected with others in the same or preceding lines, but not directly connected with more than two leading sounds or more than four sounds in all.

The reason for the restriction imposed lies in the obvious fact that beyond a certain point alliterative or assonantal play upon a particular sound is felt to be monotonous and even disagreeable. This is in accordance with the usual effect of undue repetition. The precise limits assigned are of course a matter of somewhat arbitrary choice.

In illustration of the above rule let us consider Tennyson's line already quoted in section 2. The leading sounds are evidently the sounds itali-

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cized below, where the figures 1, 2, 3, 4, 5 are set respectively above the sounds *t*, *l*, *e*, *n*, *r*; these are the only ones to enter in *aa*:

1 23 24 52 51 4 54 41 53
The league-long roller thundering on the reef.

Here the second and third sounds *t* count since they are directly connected with its initial position; all three sounds *l* are directly connected with one another and are counted; the second sound *e* (long) counts since it is directly connected with the first; the last three sounds *n* count, being directly connected with one another, but the first does not count; the four sounds *r* count since the first two are directly connected, and the last two are directly connected with the first two. Thus *aa* is 13 for this line.

A practical method of evaluation of the element *aa* is to start with the first sound to occur, and put a dot over all its positions which are counted in *aa*, then to do the same with the second sound, and thus continue to the end. The total number of dots gives the element *aa* required, at least unless it happens that the limit of desirable alliteration and assonance is exceeded.

An example of a case in which the limit of desirable alliteration and assonance is exceeded is furnished by Poe's stanza. In such rare complicated cases it is convenient to put small circles above the sounds such as *l* and *i* which are not to be counted in *aa*, but otherwise to proceed as before. Thus we obtain the following:

Virginal Lilian rigidly, humblylily dutiful;
 Saintlily, lowlily,
 Thrillingly, holily,
 Beautiful!

It is to be noted that, in accordance with our rule, the rhyming sounds in 'holily' and 'beautiful,' other than the initial *h* and *b*, are to be counted in *aa*. Furthermore the final *y* occurring five times is interpreted as a long *e* and not as a short *i* sound. We find then that *aa* is 39 in this case.

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14. THE ELEMENT *aa* OF ALLITERATIVE AND ASSONANTAL EXCESS

According to the definition of the element *aa* of alliteration and assonance, repetition of a sound beyond a certain point does not have further favorable effect. We have still, however, to make allowance for the fact that alliteration and assonance may become positively unpleasant under certain circumstances.

These circumstances are of the following three types: (1) the sound in question is a leading sound directly connected with too many earlier leading sounds; (2) the sound forms part of successive identical syllables not in the same word; (3) the sound in question is one of a uniform series of regularly recurring repetitions.

The effect produced by the immediate repetition of syllables as in (2) is cacophonous. It was to avoid this fault that Collins changed the second line* of his 'Ode to Evening' beginning

If aught of oaten stop, or pastoral song
May *hope*, *O* *pensive Eve*, to sooth thine ear —

to

May hope, chaste Eve, to sooth thy modest ear.

The element of cacophony may be compared to that of false cadence in harmony: more precisely, the immediate repetition of syllables ordinarily occurs either within a word, or in the agreeable repetition of a word; consequently when such repetition is not of one of these types, there is an unpleasant feeling of artificiality.

Likewise the effect produced by the same sound repeated several times at uniform intervals is artificial and unpleasant, whether these sounds occur in successive words, feet, lines, or stanzas.

Thus our definition of *ae* will be the number of leading sounds directly connected with more than two earlier leading sounds, and of sounds belonging to syllables immediately following the same syllables, but not in the same words, and of sounds belonging to a uniform series of the same sounds which contains at least three earlier sounds. The negative element

* See J. L. Lowes, *Convention and Revolt in Poetry* (1919).

of alliterative and assonantal excess will then be defined to be $2ae$. In other words an index -2 will be assigned to the corresponding effect.

It need hardly be said that there will rarely be found excessive alliteration or assonance of this sort in satisfactory poetry, and that when it is present, the ear will note the effect at once. Thus ae is 5 in the case of Poe's stanza because the last four ineffective elements in aa count in ae and the repeated sound ly occurs at the end of four successive words.

On the other hand, in the catch of section 7 the first line contains the sound s five times as a leading sound and the sound t four times as a leading sound, all after the first of these directly connected with the first. Hence, according to definition, two of the sounds s and one of the sounds t count in ae , so that the element $2ae$ is 6 in this line alone.

15. THE ELEMENT $2m$ OF MUSICAL VOWELS

We shall define the element of musical vowels as $2m$ where m is the number of musical vowels (a as in *art*, u as in *tuneful*, *beauty*, and o as in *ode*) increased by the number of vowels o as in *or* which are directly connected with an earlier long musical vowel o or a . The limitation will be imposed, however, that such musical vowels directly connected with more than two other earlier vowels will not be counted in m . The limitation is introduced because repetition of musical vowels is of no interest beyond a certain point.

The reason for counting the short o after the long o is merely that when one tries to pronounce the o in such a word as 'or' so that it is long, it tends to take the short form. Thus the long and short forms are closely connected, and if the long musical form of o precedes the short form and is not too far from it, the short o takes on the same musical quality. Furthermore the closeness of the two sounds justifies the similar rule for the sound a .

Such euphonious lines as

Little boy blue, come blow your horn,

and

Come into the garden, Maud,

with $2m = 8$ and $2m = 6$ respectively, show clearly the effectiveness of the musical vowels when used in this manner.

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16. THE ELEMENT *2ce* OF CONSONANTAL EXCESS

In case there are in all more than two elementary consonantal sounds for each vowel sound in any line, an appreciably harsh effect is produced. For this reason we shall define the consonantal excess as *2ce* where *ce* is the excess of the consonant sounds over twice the number of vowel sounds in each line. In illustration we consider the first line of the catch of section 7. Here there are eight vowel sounds and twenty-four consonantal sounds. Hence the excess *2ce* in this line alone is 16. The element *2ce* enters as a negative element of course.

17. THE AESTHETIC FORMULA

The complete aesthetic formula for musical quality in poetry is taken as follows:

$$M = \frac{O}{C} = \frac{aa + 2r + 2m - 2ae - 2ce}{C}$$

Here all the elements which enter have been explicitly defined in the preceding sections.

In order to evaluate *M* systematically, the following method is convenient:

(1) Determine *C* by direct phonetic analysis of each line. The successive numerals 1, 2, 3, . . . may be placed under the successive sounds of the line and under the junctures not capable of liaison.

(2) Consider the successive sounds in their order of appearance and put a dot over all those which are alliterative or assonantal, and then place a circle around those dots (if any) for which the sound does not count in *aa*.

(3) Place two additional dots over the accented vowel sounds rhyming with the same sound in an earlier position.

(4) Place two additional dots over each musical sound and a circle around those dots (if any) for which the sound does not count in *m*.

(5) The total number of dots not enclosed by circles then gives the sum *aa + 2r + 2m*.

(6) Determine the sum *2ae + 2ce*, which is in general *o* in any satisfactory poem.

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(7) Subtract this sum from that specified in (5), obtaining O .

(8) The aesthetic measure M is then O/C .

In the tabulation of these items it is convenient to put C for each line (step 1) to the right of and below the line, and O (steps (2)-(7)) to the right of and opposite each line. The ratio M giving the aesthetic measure of the line is thus displayed to the right. For a group of lines, the constituent numbers C for each line should be added to give the total C , and the constituent numbers O should likewise be added to give the total O . The aesthetic measure of the lines considered is the ratio of the total O to the total C .

18. ANALYSIS OF FIVE VERSES OF 'KUBLA KHAN'

Let us consider a particular application of this rule to the first stanza of Coleridge's 'Kubla Khan,' which affords a very remarkable example of musical quality. The analysis is indicated by the following tabulation:

$$M = \frac{87}{105} = .83$$

In	X	a	n	a	d	u	d	i	d	K	u	b	l	a	K	h	a	n		<u>22</u>
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
																				<u>21</u>

A	s	t	a	t	e	l	y	p	l	e	a	s	u	r	e	-	d	o	m	e	d	e	c	r	e	e		<u>15</u>
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22					<u>22</u>		

W	h	e	r	e	A	l	f	,t	h	e	s	a	c	r	e	d	r	i	v	e	r	,r	a	n		<u>12</u>
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22				<u>22</u>	

T	h	r	o	u	g	h	c	a	v	e	r	n	s	m	e	a	s	u	r	e	l	e	s				
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20								

t	o	m	a	n																						<u>24</u>
21	22	23	24	25																					<u>25</u>	

D	o	w	n	t	o	a	s	u	n	l	e	s	s	e	a												<u>14</u>
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15											<u>15</u>		

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These computations indicate that the number of elements of order ($O = 87$) is not much less than the number of sounds ($C = 105$), their ratio giving an aesthetic measure of .83 for the stanza. From this point of view the first, fourth, and fifth lines are the most musical.

Lamb made a very trenchant estimate of this remarkable poem in referring to it as "a vision, 'Kubla Khan,' which said vision he [Coleridge] repeats so enchantingly that it irradiates and brings heaven and elysian bowers into my parlour while he sings or says it; but there is an observation, 'Never tell thy dreams,' and I am almost afraid that 'Kubla Khan' is an owl that won't bear day-light. I fear lest it should be discovered by the lantern of typography and clear reducing to letters no better than nonsense or no sense." In other words, the significance of Coleridge's poem is elusive and slight, although the element of musical sound is almost magical. The same criticism would apply to much of Poe's poetical work of course.

19. AN EXPERIMENTAL POEM

In his essay on 'The Philosophy of Composition' of 1846, Poe analyzed step by step the construction of his poem 'The Raven' and claimed that his theory had been used as a conscious and effective tool in its composition. What is remarkable here is not that Poe had a theory. Almost every creative artist has a theory or point of view which, for him, sums up the inner secret of his success. Rather it is the fact that Poe expressed his theory in mechanical terms.

As of some interest here I shall give an account of a somewhat similar experiment made by myself on the basis of the theory described above. This experiment was undertaken in order to clarify my own ideas about the nature of poetic composition and to subject them to a test. The reader will have to judge for himself as to the success of the experiment.

According to the theory it was first of all necessary to start from an idea having some poetic quality. Here I chose an idea concerning the general nature of knowledge which I had expressed in prose as follows:*

We may compare if we will, our bits of knowledge to luminous threads which we wind into a compact, luminescent ball. By skilful arrangement of the threads

* Century magazine, June, 1929.

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there begins to appear in the center of this ball a bright vision of concepts and laws. If now we add further irrelevant threads, the vision is obscured; and if we unwind the threads in an effort to approach the vision more intimately it becomes more and more faint, and finally disappears.

My first attempts to incorporate this idea in poetical form were very unsuccessful. The chief reason for the initial lack of success seems to me now to lie in the fact that the expression of the idea was not sufficiently *terse*. The requirement of terseness is of course fundamental.

Then one day came without apparent effort the following:

VISION

Wind and wind the wisps of fire,
Bits of knowledge, heart's desire;
Soon within the central ball
Fiery vision will enthrall.
Wind too long or strip the sphere,
See the vision disappear!

The aesthetic measure of this short poem according to the criterion above is .62. Comparison with the ratings of an arbitrarily selected list of poetic lines (section 20) indicates that the poem may be considered as of good musical quality according to our theory. In this case the poetical form of expression, although more terse, falls short of the prose form in exactitude, but has perhaps the advantage of inducing more emotional interest. In the writing of these six lines there was certainly no conscious use of the formula. Nevertheless I believe I could not have done nearly so well without conscious reflection concerning the aesthetic factors in musical quality taken account of by the formula.

20. FURTHER EXAMPLES

For the purpose of testing the theory a number of characteristic opening lines of varying musical quality were selected, and then arranged by others in the order of their aesthetic preference, as far as musical quality was concerned. The arrangement thus obtained was found to be substantially in accord with that indicated by the theory as tabulated below.

$$M = .83$$

In Xanadu did Kubla Khan
A stately pleasure-dome decree:

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Where Alf, the sacred river, ran
Through caverns measureless to man
Down to a sunless sea.
From Coleridge's 'Kubla Khan'

M = .77

Come into the garden, Maud,
For the black bat, Night, has flown,
Come into the garden, Maud,
I am here by the gate alone;
And the woodbine spices are wafted abroad,
And the musk of the roses blown.

From Tennyson's 'Maud'

M = .74

Take, O, take those lips away,
That so sweetly were foresworn;
And those eyes, the break of day,
Lights that do mislead the morn!

From Shakespeare's song
'Take, O, Take Those Lips Away'

M = .73

Tell me not, in mournful numbers,
Life is but an empty dream! —
For the soul is dead that slumbers,
And things are not what they seem.

From Longfellow's,
'A Psalm of Life'

M = .65

Little boy blue, come blow your horn,
The sheep's in the meadow, the cow's
in the corn.

From a nursery rhyme

M = .64

The white mares of the moon rush along the sky
Beating their golden hoofs upon the glass Heavens;
The white mares of the moon are all standing
on their hind legs
Pawing at the green porcelain doors of the
remote Heavens.

From Amy Lowell's
'Night Clouds'

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$M = .62$

Bright Star, would I were steadfast as thou art —
From Keats' 'Last Sonnet'

$M = .57$

Hear the sledges with the bells —
Silver bells!
What a world of merriment their melody
foretells!
From Poe's 'The Bells'

$M = .51$

Onward, Christian soldiers,
Marching as to war,
With the cross of Jesus
Going on before.

From Baring-Gould's
'Onward Christian Soldiers'

$M = .45$

He never had much to give,
Subscription lists knew not his name,
He was one of the many who live
Unrecorded in charity's fame.

From E. A. Guest's
'Contribution'

It may be remarked that instances of consonantal excess appear in three of these cases, namely in the short second line of Poe's 'The Bells' ($ce = 1$), in the first line of Baring-Gould's 'Onward, Christian Soldiers' ($ce = 6$), and in the second line of Guest's 'Contribution' ($ce = 4$). As is almost obvious, the two opening lines from 'The Bells' are not among the best of Poe's in musical quality; for example, the first stanza from 'The Raven,' has an aesthetic measure M of .75. Furthermore in the free verse of Amy Lowell the 'accented' syllables were determined by ear, and the third and fourth lines were taken as 'rhyming' with the first and second respectively, because of the repetition of words.

It is well to bear in mind the precise significance claimed for these and similar results:

THE MUSICAL QUALITY IN POETRY

(1) The aesthetic measure M defined above is applicable primarily to the great body of English poetry of conventional type; the definition made can doubtless be considerably improved on the basis of further experiment.

(2) Only the musical quality of this kind of poetry is so measured. Every good poet will find it desirable to sacrifice this musical quality occasionally in order to produce some subtle musical effect or to increase expressiveness.

(3) In so far as this measure M is applied to more recent writers (such as Amy Lowell, for instance), it serves only as an indication of the presence or absence of musical quality of this conventional type.

21. SONOROUS PROSE

It is obviously possible to measure the musical quality of sonorous prose by the same methods. For this purpose it is only necessary to write the prose as nearly as possible in the form of verse and then to apply the same rules. For instance the following sentence from Sir Thomas Browne's 'Hydriotaphia' is so written:

Circles and right lines limit and close all bodies,
And the mortal right-lined-circle
Must conclude and close up all.

As written, the sentence has an aesthetic measure M of .61 and so must be regarded as on a level with much poetry in degree of musical quality.

22. POETRY IN OTHER LANGUAGES

As far as I have been able to make out, the aim of poetry is essentially the same, whatever the language or period. It is true that rhyme may assume different forms or may be absent as in blank verse. But the fundamental aim is always to achieve the terse, imaginative expression of a poetic idea in metric form by use of language of unusual musical quality.

23. THE RÔLE OF MUSICAL QUALITY IN POETRY

In order to avoid misunderstanding I would like to emphasize once more that musical quality is only one of the essential elements in poetry, and that even this quality cannot be measured in its more delicate *nuances*.

AESTHETIC MEASURE

by any mechanical method, such as that given above. At the same time, it seems to me that some such objective method of evaluation can play a useful if modest rôle.

Paul Valéry, the French poet and critic, has expressed the extraordinary difficulty of poetic achievement: *

One feels clearly in the presence of a beautiful poem of some length how slight the chance is that a man could have improvised without revision, without other fatigue than that of writing or uttering what comes to his mind, an expression of thought, singularly certain, showing power in every line, harmonious throughout, and filled with ideas that are always felicitous; an expression that never fails to charm, in which there are no accidents, no marks of weakness or of lack of power, in which there are no vexatious incidents to break the enchantment and destroy the poetic universe.

Nevertheless, in achieving this complex, difficult end, the poet must take cognizance of the essential formal factors of metre and musical quality which differentiate poetry from prose. Of this necessity upon the poet, and of the others, Valéry speaks as follows:

Behold the poet at grips with this unstable and too mixed material [of language]; constrained to speculate concerning sound and sense in turn, to achieve not only harmony and musical phrasing, but also to satisfy a variety of intellectual conditions, logic, grammar, the subject of the poem, figures and ornaments of all kinds, not to mention conventional rules. See what an effort is involved in the task of bringing to a successful end an expression of thought in which so many demands must all be miraculously satisfied!

* 'La Poésie,' Conferencia, Paris (1928), my translation.